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FOREWORD

Chapters I to IV and Chapter VI of this book have already appeared, in a somewhat different form, as articles in the following: *Platon et Aristote à la Renaissance* (XVI[e] Colloque International de Tours), 1976; *Proceedings of the Royal Musical Associaton*, Vol. 100, 1974; *Journal of the Warburg and Courtauld Institutes*, Vol. XXX, 1967; *Music and Letters*, Vol. 57, 1976. Chapters V, VII and VIII will probably have appeared before this book is published in: *La Chanson à la Renaissance* (XX[e] Colloque International de Tours); *Archives Internationales d'Histoire des Sciences*; papers of the Oxford International Symposium on Musicology (1977).

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September 1977

CHAPTER I

THE HARMONY OF THE SPHERES

The purpose of this chapter is to survey a wide and ancient tradition in order to indicate fruitful lines of research within it. The exploration of some of these has already been begun by modern scholars, and a few of them I have followed up in the remaining chapters of this book.

This tradition, namely the harmony of the spheres, the *musica mundana* of Boethius, has hitherto been chiefly studied as a literary one, a complex of metaphors and topoi, very thoroughly and brilliantly covered by James Hutton in a long article entitled, too modestly, "Some English Poems in Praise of Music" [1]. I want now to try and see when, where and in what ways *musica mundana* has had some importance outside literature, some reality as a part of, or influence on the following fields: ordinary music, astronomy and cosmology, astrology and magic, architecture, mathematics and early modern science.

In this tradition we can distinguish two strands, closely entwined together already in Plato:

1) numerology, number-mysticism.

2) music as a science, the ideal of a mathematical universe. The two strands clash and begin to separate in the seventeenth century, for example, in the controversy between Robert Fludd on the one hand, and Kepler, Mersenne and Gassendi on the other, on which there is a good essay by Pauli [2], though certainly much remains to be done. This is no doubt an over-simplified

[1] James Hutton, "Some English Poems in Praise of Music", *English Miscellany*, ed. Mario Praz, II, Rome, 1951, pp. 1-63; cf. John Hollander, *The Untuning of the Sky*, Princeton, 1961; G. L. Finney, *Musical Backgrounds for English Literature: 1580-1650*, New Brunswick, n.d.

[2] C. G. Jung & W. Pauli, *The Interpretation of Nature and the Psyche*, London, 1955.

framework, but it may be better than nothing as a guide through the bewilderingly huge survey I have embarked on.

It is evident that of the two strands the first is to us the most puzzling, the most foreign to our ordinary habits of thought, and, precisely for this reason, I think it most important not to neglect it, not to dismiss it contemptuously, before we have made at least a serious effort to understand it, to make some kind of sense out of it. We may of course end up with a negative result, a risk that must be run in any genuinely exploratory research. Perhaps large areas of numerology were just an aimless, childish and boring game which consists in collecting sets of things whose only common characteristic is having the same number of members—the four elements, the four points of the compass, the four evangelists, etc. But even so, we still have to account for the remarkable persistence of numerology, and for the interest taken in it by so many thinkers, some of them of the highest importance—to name only a few from antiquity and the Renaissance: Plato (the terrible nuptial number in the *Republic*, VIII) [3], Macrobius (Commentary on the *Somnium Scipionis*) [4], Francesco Giorgi [5], Robert Fludd [6], and even Leibniz. And there is also the vast field of mediaeval number-mysticism, which I know about only second-hand, through the works of V. H. Hopper, H. Abert and Huizinga [7].

Moreover, there are at least two directions in which numerology does lead to something of interest and value:

1) arithmetical speculations about different kinds of number: square, cube, triangular, prime, perfect, etc.

2) the importance of certain numbers in Neoplatonic metaphysics and Christian theology: God as the One and the Good,

[3] Plato, *Republic*, 546 A-D; cf. Paul Tannery, *Mémoires scientifiques*, I, 12 seq.; Ficino, *Opera Omnia*, Basle, 1576, pp. 1413 seq.

[4] Macrobius, *Comm. in Somn. Scip.*, II, i-iv.

[5] Francesco Giorgi, *Harmonia Mundi*, Venice, 1525.

[6] See P. J. Ammann, "The Musical Theory and Philosophy of Robert Fludd", *Journal of the Warburg & Courtauld Institutes*, Vol. XXX, 1967, pp. 198-227.

[7] V. H. Hopper, *Mediaeval Number Symbolism*, New York, 1938; H. Abert, *Die Musikanschauung des Mittelalters*, Halle, 1905; Huizinga, *The Waning of the Middle Ages*, London, 1950, Ch. XV.

Proclus' henads, Leibniz's monadology, Neoplatonic triads of principles, the Christian Trinity as, for example, in St. Augustine's *De Trinitate*, and on a higher mathematical level, the geometrical analogies of Nicolas of Cusa and Kepler. This trend eventually, in the seventeenth and eighteenth centuries, with the advent to the numbers of zero and the ability to deal mathematically with different kinds of infinity, produced some very interesting theologico-metaphysical speculation: Leibniz finding the *creatio ex nihilo* in his binary numbers [8]; or the Newtonian George Cheyne's use of different kinds of infinity in his natural theology [9]. Perhaps one might add to these directions the recent burst of scholarship which, deriving from a few remarks in Curtius' *Lateinisches Mittelalter* [10], finds numerological patterns in mediaeval and Renaissance poetry. I am a little suspicious of this.

The connexions of the Harmony of the Spheres tradition with some of the subjects I have mentioned are obvious enough: cosmology and astronomy, from Plato's *Timaeus* to Kepler's *Harmonice Mundi*; but others are less evident, and I want now to indicate what they are.

Connexions of a certain kind with ordinary music are quite easy to see. The Harmony of the Spheres has a firm place in the literary tradition of the *Laus Musices*, and such encomia appear regularly near the beginning of ordinary treatises on the theory of music. But what I have in mind are less purely metaphorical links than those made through highly generalized and rather vague concepts of harmony and proportion—links that are more precise and may lead either to practical musical results or to theories of consonance and explanations of the emotional power and meaning of music. The traditional Praise of Music also always contained examples of the marvellous effects of ancient

[8] See D. P. Walker, *The Ancient Theology*, London, 1972, pp. 223-4.

[9] George Cheyne, *Philosophical Principles of Religion: natural and revealed: in two parts . . . Part II. Containing the nature and kinds of Infinites*, London, 1715.

[10] E. R. Curtius, *Europäische Literatur und Lateinisches Mittelalter*, Bern, 1948, pp. 493-500.

music, such as the story of Timotheus and Alexander [11]. Both these themes, *musica mundana* and the *maravigliosi effetti della musica antica*, provide a good example of an important historical phenomenon: a petrified literary tradition, transmitted through many centuries by sheer copying, may suddenly become alive, be taken seriously and practically, as the Effects of Music were by some sixteenth-century humanists, such as Baïf's Academy [12] and the Florentine Camerata [13], and as the Harmony of the Spheres was by Ficino [14] and Kepler [15]. How dead as well as petrified this literary tradition became even in antiquity can be seen in Macrobius' *Comm. in Somn. Scip.* Though he faithfully and correctly transmits the ratios of the Pythagorean scale, he does not even understand that they *are* ratios; he states that you cannot divide a tone, 9:8, in half because 9 is not divisible by 2 into two integers [16]—the real difficulty of course is that there is no rational square root of 8.

To return to these links between our tradition and practical music:

First, the Harmony of the Spheres may itself be the subject of a piece of music, as it was of the Florentine *Intermedii* of 1589 [17], or it may play a major rôle in a musical-dramatic work such as the *Balet comique de la Royne* of 1581 and the other festivities at this wedding [18].

Secondly, by way of astrology, our tradition was active in Ficino's attempt to create magically powerful songs, his Orphic singing, and in later versions of his magic, such as Paolini's, who applied it to oratory, or Campanella's, in those astrological rites

[11] See Hutton art. cit. supra n. 1, and D. P. Walker, "Musical Humanism in the sixteenth and Early seventeenth Centuries", *Music Review*, 1941-2.

[12] See F. A. Yates, *The French Academies of the Sixteenth Century*, London, 1947, and *Musik in Geschichte und Gegenwart*, art. "Baif".

[13] See *MGG*, art. "Camerata".

[14] See D. P. Walker, *Spiritual and Demonic Magic*, London, 1958, pp. 14 seq.

[15] V. infra Ch. IV.

[16] Macrobius, *Comm. in Somn. Scip.*, II, i, 20-2.

[17] *Musique des Intermèdes de la "Pellegrina"*, ed. D. P. Walker, Paris, C.N.R.S., 1963.

[18] See F. A. Yates, "Poésie et musique dans les 'Magnificences' au mariage du duc de Joyeuse, Paris, 1581", *Musique et Poésie*, Paris, C.N.R.S, 1954.

he secretly performed with Pope Urban VIII [19]. We should remember also Ficino's quite impressive *spiritus* theory of the power of music [20]; and someone should look seriously at his commentary on the *Timaeus*, from which it appears that he had read in manuscript Ptolemy's *Harmonica* [21].

Thirdly, the mathematical and astronomical side of the tradition, the effort to make a precise correlation between the ratios of musical intervals and the distances, speeds or orbits of the planets, led in at least one case, Kepler, to interesting and original explanations of the emotional power of music and to support for the practical use of a certain system of intonation, namely, just [22]. It is through this question of intonation, to which I will return, that there are important connexions with science, other than the astronomical and cosmological ones, and of course with ordinary music.

Other connexions with science result from the fact that there were opponents of the Harmony of the Spheres tradition, thinkers who rejected the whole mathematical basis of musical intervals, and proposed a purely empirical investigation of the causes of consonance and dissonance, and an empirically established musical scale: among the ancients Aristoxenos [23], among the moderns Bacon and Campanella [24], and certainly others I do not know about. An examination of these contrasting points of view and of controversies arising from them, mentioned by John Dee in his famous Preface to Billingsley's *Euclid* (1570) as disputes between "Harmonists and Canonists" [25], would surely throw light on the relationship in early modern science between empiricism and more or less *a priori* mathematical theory. Here some work has

[19] See D. P. Walker, *Magic*, pp. 204-212.

[20] Ibid., pp. 5-11.

[21] Ficino, *Comm. in Tim.*, in *Op. Omn.*, pp. 1456-7; cf. Kristeller, *Supplementum Ficinianum*, Florence, 1937, I, 51-4.

[22] V. infra Ch. IV.

[23] See H. S. Macran, *The Harmonics of Aristoxenos*, 1902, and *MGG*, art. "Aristoxenos".

[24] V. infra Ch. VII, and Walker, *Magic*, pp. 201, 231.

[25] John Dee, "Mathematicall Preface" to *The Elements of Geometrie of . . . Euclide*, tr. Sir Henry Billingsley, London, 1570.

already been done: there is Barbour's excellent history of tuning and temperament [26], and there are articles by Crombie [27] and Palisca [28]. But there is a great deal more to do.

Another of the subjects I mentioned earlier has some close links with celestial and terrestrial music: architecture. First, some Renaissance buildings were designed on musical proportions, as Wittkower has shown [29]; and we have Francesco Giorgi's musical plan for the church of San Francesco della Vigna at Venice [30]. Secondly, the chapters in Vitruvius on music are Aristoxenian, that is, in opposition to our tradition, and at least one of his Renaissance commentators, Daniele Barbaro, is sharply critical of Vitruvius and in his commentary gives an orthodox Pythagorean musical treatise [31]. It would be well worthwhile looking at other commentaries on Vitruvius, of which there are a great many.

The connexions between musical theory and mathematics begin very early, certainly before Plato, with the discovery of the ratios of the perfect consonances, traditionally attributed to Pythagoras. Paul Tannery [32] argues convincingly that, though some theory of ratios may be more ancient than this, there are very early signs of the influence of musical problems on Greek mathematics, for example, in Euclid V and VI squared or cubed ratios are said to be doubled or tripled, which would be a natural terminology in a musical context, where adding two equal intervals involves squaring their ratios (5th + 5th, or $2 \times 5\text{th} = \left(\frac{3}{2}\right)^2$). With the discovery of irrational quantities, also attributed to the Pythagoreans, come other probable cases of musical

[26] J. M. Barbour, *Tuning and Temperament*, East Lansing, 1953.

[27] A. C. Crombie, "Mathematics, Music and Medical Science", *Organon*, No. 6, 1969, pp. 21-36.

[28] C. V. Palisca, "Scientific Empiricism in Musical Thought", *Seventeenth Century Science and the Arts*, ed. H. H. Rhys, Princeton, 1961.

[29] R. Wittkower, *Architectural Principles in the Age of Humanism*, London, 1949.

[30] See Wittkower, op. cit., pp. 90 seq.

[31] D. Barbaro, *De Architectura libri decem, cum commentariis*, Venice, 1567.

[32] P. Tannery, "Du rôle de la musique dans le développement de la mathématique pure", in Tannery, *Mémoires scientifiques*, III, 69-89.

influence on mathematics, such as the early invention of methods of successive approximation to $\sqrt{2}$ or other surds, which is more likely to come from efforts to divide the octave or tone in half than from architects or other technicians, who could so easily construct such irrationals geometrically. It is by the way worth bearing in mind that the Greek word for a ratio is λογος. For thinkers like Kepler, or a little later Meibom [33], who believed that geometric ideas are coeternal with God, the beginning of the Gospel of St. John may have had a strange meaning: ἐν ἀρχῃ ἦν ὁ λογος.

This early discovery of musical ratios was unique and momentous, one of the main starting-points of Greek mathematically orientated science. The discovery meant that an immediately given, subjective, sensible quality was found to be exactly correlated with measurements expressible as simple numerical ratios, all having the same pattern, superparticular ($n + 1 : n$); and this correlation was established empirically by using the monochord. We feel whether an octave or a fifth is in tune or not, and with great accuracy; we get them exactly in tune, then we measure the strings on our monochord, and lo! they are as 1:2, 2:3. The whole world of sensible qualities, e.g. hot, cold, dry, wet, might then be explicable in the same way; the ultimate elements, the basic structure of the physical universe might be found to show similar, though not necessarily identical, simple elegant mathematical ratios.

Now the other, still more ancient body of well-established, mathematically expressible observations was of course astronomy. Here we do not have the extraordinary fact of sensible (secondary) qualities corresponding to mathematical ratios, nor is the geometry involved all that simple and elegant. But still, as long as one could "save the phenomena" by means of the simplest and most beautiful figure, the circle, the situation was not too bad, even if the planets needed a lot of epicycles. It was natural, then, to bring the

[33] Marcus Meibomius, *De Proportionibus Dialogus*, Hafniae, 1655.

music and astronomy together, and hence we have the harmony of the spheres already in Plato's *Timaeus* [34] and *Republic* [35].

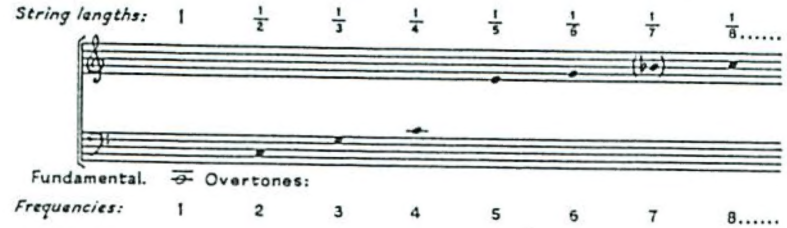
The musical ratios which Pythagoras perhaps discovered and which appear in the *Timaeus* are those of the so-called Pythagorean scale, in which all the fifths are just (3:2), there are only major tones (9:8), the semitones are narrow (256:243), and the thirds and sixths are dissonant (81:64; 27:16). This system prevailed, at least in theory, until the sixteenth century, when it gave way to just intonation. The latter is a scale having the maximum number of just consonances. If all the consonances were just, the scale would be hopelessly unstable even in the simplest diatonic music, e.g. C, f, a, d, g, C, ends flat by a comma (81:80); or in other words, it is impossible to construct a diatonic scale in which all the consonances are just. Hence the usual form of the scale is one which has a narrow fifth, D to a in the scale of C, and a narrow minor third, D to f. It has major and minor tones (9:8 and 10:9), wide semitones (16:15), and, with the above two exceptions, all the consonances are just, i.e., are intervals given by the natural series of overtones. But it was invented long before anything was known of overtones; for it is already in Ptolemy's *Harmonica* [36], and the scale was often called after his name. The system of just intonation was reached by purely mathematical means, though the sweetness of the consonances was confirmed empirically by the monochord. By using harmonic proportion (if $a > b > c$, $\frac{a}{c} = \frac{a-b}{b-c}$; the harmonic mean between a and c, $b = \frac{2ac}{a+c}$) one can divide the octave into a fifth and a fourth, the fifth into a major and a minor third, the major third into a major and a minor tone.

[34] Plato, *Timaeus*, 34 B seq. Here the harmony of the spheres is not actually mentioned, but is strongly suggested since, after the musical construction of the *anima mundi*, its strips are bent into an armillary sphere.

[35] Plato, *Republ.*, 6161 D seq.

[36] Ptolemy, *Harmonica*, Lib. I, c. xv.

The harmonic series: 1, ½, ⅓, ¼ . . . also points directly to the overtone series; for this is how a string does in fact divide itself up into vibrating parts, thus producing the overtones, or partials:



It is, I think, surprising that the overtones were discovered so late; one would have thought that anyone just fiddling about with a bowed instrument, let alone playing a trumpet or a tromba marina, would have hit on them. But Descartes, in his *Compendium Musicae*, written in 1618, is only on the brink of the discovery; he notes that the long-known phenomenon of sympathetic vibration also occurs with strings tuned, not in unison, but in an octave or a twelfth [37]. In the *Harmonie Universelle* (1636) Mersenne has observed higher partials [38]; but even towards the end of his life he is still uncertain about them and puzzled by them—in 1647 he writes to Constantijn Huygens asking him for help in clearing up these problems [39]. And as late as 1675 Christiaan Huygens does not get the series quite right: he gives a tenth, instead of a third plus two octaves, for the fifth partial [40].

These two systems, which were in competition during the sixteenth century, Pythagorean and just, are of course mathematical ideals to which musical practice corresponds only very roughly. Nevertheless it does matter which mathematical model one aims at, and the resultant differences in practice are perceptible [41]; hence the enormous amount written about them in

[37] Descartes, *Oeuvres*, ed. Adam & Tannery, XI, Paris, 1908, pp. 97, 99, 103.

[38] See Hellmut Ludwig, *Marin Mersenne und seine Musiklehre*, Berlin, 1935, pp. 40 seq.

[39] Chr. Huygens, *Oeuvres*, I, 59-60.

[40] Ibid., XIX, 366-7.

[41] To convince himself that he can hear the difference between just and Pythagorean thirds and sixths, the reader who owns a violin or 'cello may make the following

the sixteenth and seventeenth centuries. Moreover, instruments with fixed intonation, i.e. keyboard of fretted, cannot use either of these systems, but must use some kind of temperament. Here again there were two competing methods: various kinds of mean-tone temperament and equal temperament; but the latter was a necessity for fretted instruments, and the former was almost universally recommended for keyboard instruments. Both kinds of temperament involve ratios including irrational numbers. Mean-tone is based on the principle of dividing just major thirds (5:4) into two equal tones, which therefore have the ratio $\sqrt{5}:2$; such scales have sweet thirds and sixths and narrow fifths. Equal temperament is a division of the octave into twelve equal semitones of the ratio $\sqrt[12]{2}:1$, so that all the intervals are slightly false—the fifths a little narrow and the major thirds a little wide. Aristoxenos had proposed long ago dividing the scale into equal parts; but this, apparently, was to be done by ear, which is not at all easy. By the later sixteenth century theorists were capable of dealing mathematically with the problems of temperament. It is evident that once logarithms were invented there was a very easy method of obtaining close approximations to these irrationals; and it is another surprising fact that, although Napier's first work on logarithms was published in 1614 [12], they were not used for this purpose until the second half of the century [13].

We should bear in mind that irrational quantities belong to what was once perhaps an awe-inspiring, mysterious side of mathematics. Think, seriously, of a non-recurring, interminable decimal, i.e. one of the orthodox definitions of an irrational quantity; of series that approach ever nearer to a surd, but which

simple experiment. Having tuned the instrument as accurately as possible, play E on the D-string with the open G-string; then, taking care not to move your finger, play the E with the open A-string. If the major sixth has been made as sweet as possible, it will be found that the finger has to be leaned considerably forward to produce a perfect fourth. The difference between the two E's is a comma (81:80). Then try the experiment the other way round.

[12] Napier, *Logarithorum Canonis descriptio*, Edinburgh, 1614; a description of Napier's logarithms was published in French in Paris in 1624 (see Mersenne, *Correspondance*, I, 314).

[13] Cf. infra Ch. VII.

we know can never reach it; or of infinite series the sum of which converges towards a rational number; or again of the infinite quantity of rational numbers between any two rational numbers, and so forth [44]. Towards the end of the sixteenth century the mathematician was beset on all sides with different kinds of infinity, not only the infinitely great universe [45], and the infinitely small universe if matter is indefinitely subdivisible, but also all these infinite series trailing away eternally towards a number or a surd, and never getting there; whereas previously infinity and eternity had been strictly confined to God. One would therefore expect that the people who first fully realized and accepted that a man-made system was riddled with these pits of infinite depth must have been thinkers of wildly bold, soaring imagination and were likely to be of a deeply religious, mystical cast of mind. And this expectation is in fact borne out: Pascal poising man between the infinitely great and the infinitely small (and "le silence éternel de ces espaces infinis m'effraie") [46]; Napier commenting on the Apocalypse [47]; Kepler worshipping his geometrizing God and using the most extraordinary sexual metaphors [48]; Leibniz finding the creation in his binary numbers [49]; Newton, a secret Arian, copying out mystical alchemical treatises [50]. As Descartes remarked to Mersenne, "la partie de l'esprit qui aide le plus aux Mathématiques, à sçavoir l'imagination, nuit plus qu'elle ne sert pour les spéculations métaphysiques" [51]. There is in Plato, the main starting-point of this whole tradition, the same combination: poetic imagination and religious feeling, coupled with an intense interest both in genuine mathematics and numerology. This, I think, is one explanation of why number-mysticism and

[44] Cf. Friedrich Waismann, *Einführung in das mathematische Denken*, 3rd ed., Munich 1970.

[45] Cf. A. Koyré, *From the Closed World to the Infinite Universe*, Baltimore, 1957.

[46] Pascal, *Pensées*, ed. Brunschvig, Nos. 72, 206.

[47] Napier, *A Plaine Discovery of the whole Revelation of Saint John*, Edinburgh, 1593.

[48] V. infra Ch. IV.

[49] V. supra p. 3, n. 8.

[50] See B. J. T. Dobbs, *The Foundations of Newton's Alchemy*, Cambridge, 1975.

[51] Mersenne, *Corresp.*, VIII, 611.

proper mathematics remained so long entwined together. We can contrast this type of mind with someone like Montaigne, who was basically sceptical and not inclined to religious awe; when Jacques Péletier told him about asymptotic curves, he drew the conclusion that geometry, then considered the most certain of sciences, was therefore unsound [52].

I shall end this chapter by briefly discussing theories of consonance and dissonance. These are a particularly interesting and fertile field of research because they are a cross-roads where several different aspects of our tradition meet: the practice and history of music, aesthetics, mathematics, the scientific investigation into the nature of musical sound. I shall just outline the main problems involved and the main kinds of theory that were put forward from the sixteenth to the eighteenth century.

The problems are ones that we are still wrestling with, and they arise because we are dealing at the same time with an historical tradition, the evolution of harmonic language in western music, and also with some unchanging acoustical facts, such as the natural series of overtones and difference-tones. That this series is relevant to harmony is undeniable; one just cannot accept as a pure coincidence that the first five partials give a perfectly spaced major chord. But how important this fact has been as a shaping influence, how to explain the minor mode, the possible relevance of the seventh partial to the chord of the dominant seventh, why the $\frac{6}{4}$ chord is treated as a dissonance—all these are questions to which there are no simple or certain answers. As I have mentioned, the existence and nature of the overtone series was only gradually being discovered during the seventeenth century by scientists such as Mersenne, Huygens and Wallis, but its mathematical equivalents, the harmonic series 1, $\frac{1}{2}$, $\frac{1}{3}$ . . . . $\frac{1}{6}$ and the just ratios of the consonances, were firmly established from the time of Zarlino onwards.

There were three main types of theory of consonance at this period:

[52] Montaigne, *Les Essais*, ed. Villey, Paris, 1922, t. II, p. 327 (II, xii).

1) A purely arithmetic theory, such as Zarlino's, based on the mathematically harmonic series and the fact that the ratios of the consonances are contained within the *senario*, the numbers 1 to 6 (with the annoying exception of the minor sixth). This type of theory had the weaknesses that it had, at that time, no physical basis, and that it could offer no psychological explanation of how the mind could immediately be aware of these ratios. When the existence of overtones was fully established, any such theory was strongly reinforced, as actually happened in the case of Rameau, whose first book on musical theory was published before he had heard of Sauveur's work [53]. But the purely arithmetic theory persisted in Italy into the late eighteenth century with theoreticians such as Vallotti and Padre Martini, who consciously rejected any physical explanations [54].

2) Theories based on geometry. These are very rare. I know of only two examples: Kepler, and, much later and independently of him, Giuseppe Tartini, whose first work on musical theory was published in 1754 [55].

3) A theory based on the coincidences in the vibrations of two or more sound-waves. Consonances are graded so that the greatest frequency of coincidence causes the highest degree of consonance. This produces the traditional hierarchy of consonances, in descending order: octave, fifth, fourth, major third, minor third. This type of theory was the prevailing one in our period, and also in the eighteenth century. The first expounder of it, as far as I know, was Benedetti in 1585, and among its adherents were: Beeckman, Descartes, Galileo Galilei, and Huygens. It had the advantage of being compatible with our first type, and also, as we shall see when looking at Galilei's theories, of offering a psychological explanation of the perception of musical ratios. The great weakness of the theory seems not to have been noticed, namely, that it cannot account for the acceptability of tempered consonances, whose vibrations, after the first pulse, will never coincide [56].

[53] See M. Shirlaw, *The Theory of Harmony*, London, n.d., p. 134.
[54] V. infra Ch. VIII, p. 125.
[55] V. infra Ch. IV and VIII.
[56] V. infra Ch. III.

CHAPTER II

VINCENZO GALILEI AND ZARLINO

In this chapter I want to bring out two points which are, I hope, of interest not only to musicologists, but also to historians of ideas in other fields. The first of these is that polemical writings often present to the historian peculiar difficulties of interpretation, especially when the two adversaries, on the one hand, genuinely hate each other, and, on the other, may in fact agree on the main subject under discussion. This was, I believe, the case in the controversy between Vincenzo Galilei and Zarlino, and the resultant dishonesty and evasion of crucial problems makes very tricky the task of disentangling their true thought.

The other general point concerns the relation in early modern science of mathematical schemes to physical and psychological realities. I am thinking of the acceptance of just intonation, as systematized by Zarlino [1], well before even the first steps had been taken towards the observation, let alone understanding, of the natural series of overtones: that is to say, a simple mathematical scheme was widely accepted as the basis of theories of consonance and of an ideal of musical practice, and this scheme was only considerably later, and very slowly, found to fit perfectly the empirically observable behaviour of vibrating strings and columns of air. But this perfect fit was not of course a sheer fluke, an improbably happy accident. Right from the start, from the pre-Platonic discovery of the ratios of the perfect consonances, the mathematical scheme had been empirically verified by the use of the monochord. When the mathematical scheme was elaborated without the empirical check, it went wrong, as we shall see.

[1] Zarlino, *Istitutioni Harmoniche*, Venice, 1558. Ramis de Pareia (*Musica Practica*, Bologna, 1482) and Foligno (*Musica Theorica*, Venice, 1529) had already both given the ratios of just consonances.

The controversy between Vincenzo Galilei and Zarlino had its ultimate origin, as Claude V. Palisca has shown [2], in the correspondence between Galilei and Girolamo Mei, and its central subject was the system of intonation used in contemporary *a capella* singing. Until this correspondence began, in 1572, Galilei seems to have accepted his teacher's, Zarlino's, original view that the system used was Ptolemy's diatonic syntonon, i.e. a scale having just consonances except for a narrow fifth and minor third on the supertonic. Mei, in replies to a query of Galilei in a letter now lost, pointed out, first, that mathematical theory and musical practice were unlikely to coincide exactly, since the ear could tolerate considerable divergences from any mathematically exact scale; secondly, that it was more likely that singers aimed at Pythagorean intonation, which had been the only one advocated by the great majority of theorists before Zarlino; and, finally, in a later letter, he suggested that Galilei should test this empirically by tuning two lutes in the two scales and then comparing intervals on them with those actually sung. Whether Galilei took this sensible advice we do not know; but at all events his loyalty to Zarlino's view was shaken, and in 1578 he sent him, anonymously, a discourse on the subject [3], now unfortunately lost, which presumably already contained some of the criticisms of Zarlino later published in Galilei's *Dialogo della musica antica et moderna* (1581) [4].

Here began the animosity between the two musicians, which was further exacerbated by Galilei's suspicion that Zarlino had deliberately held up the publication of the *Dialogo* at Venice [5].

[2] Girolamo Mei, *Letters on Ancient and Modern Music*, ed. Claude V. Palisca, American Institute of Musicology, 1960, pp. 63-9. Palisca has published an article (cited above Ch. I, p.6, n. 28) which covers much of the same ground as this chapter; but our points of view are radically different. The same is true of an article, largely based on Palisca's, by Stillman Drake, "Renaissance Music and Experimental Science", *Journal of the History of Ideas*, XXXI, 1970, 483-500.

[3] Mei, op. cit., p. 67.

[4] Florence, 1581 (fascimile ed., New York, 1967).

[5] See *Dialogo*, dedication to G. Bardi; and V. Galilei, *Discorso intorno alle opere de Gioseffo Zarlino et altri importanti particolari attenenti alla musica*, Venice, 1589 (facs. ed., Milan, 1933), p. 14.

In Zarlino's *Sopplimenti*[6] of 1588 and in Galilei's *Discorso intorno alle opere di Gioseffo Zarlino*[7] of 1589 and in his later treatises, still in manuscript[8], the animosity has blossomed into what was, I think, genuine hatred of an intensity which made it difficult or impossible for either party to concede the validity of any theory or statement of the other, or of course, conversely, the invalidity of any of his own statements. Zarlino's tone is quieter and more ironical (he usually calls his adversary "il mio amorevole Discepolo"), and Galilei's more violent and openly insulting; but the resultant malicious distortion of the adversary's thought is almost as great in the former as the latter. For example, Zarlino, when discussing Galilei's simple method of fretting a lute in equal temperament by shortening the string successively by an eighteenth part, laboriously works out (17:18)[12] in order to show that the result is not exactly 1:2, although Galilei was plainly aware that his method was only an approximation, though a very close one[9].

But more serious for the historian than such deliberate misunderstandings are the consequences of the basic agreement between the two, coupled with the desperate wish to contradict each other. The dilemma in which both Zarlino and Galilei found themselves was the following. If unaccompanied singers use just intonation, they have two alternatives: either they must learn to sing a narrow third and fifth on the second degree, narrow by a comma of Didymus (81:80), as in Ptolemy's scale; or, using only just intervals, they must accept extreme instability of pitch, even in strictly diatonic music[10]. In the *Sopplimenti* Zarlino chose the second horn of the dilemma, that is, he claimed that singers did not keep to Ptolemy's scale but sang only just consonances on all degrees; he called Ptolemy's scale the "sintono

[6] Gioseffo Zarlino, *Sopplimenti musicali*, Venice, 1588.
[7] V. supra note 5.
[8] Florence, Biblioteca Nazionale, MSS. Galileiani, Anteriori a Galilei, vols. 1-8; cf. *MGG*, Palisca's art. Galilei, Vincenzo.
[9] Zarlino, *Sopplimenti*, pp. 204-5; Galilei, *Dialogo*, pp. 49-55, and *Discorso*, p. 55; cf. infra Ch. VII, p. 116.
[10] Cf. J. M. Barbour, op. cit., pp. 196-9, and above Ch. I, p. 8.

artifiziale" and the unstable, purely just intonation the "sintono naturale"[11]. He dealt with the problem of instability by avoiding all mention of it whatever.

Galilei, in the *Dialogo*, was not yet caught in the dilemma, since, with sensible tentativeness and vagueness, he there suggested that singers used a scale approximating to mean-tone temperament[12], a guess which may well have been correct, as singers were so often accompanied on keyboard instruments. But in the *Discorso* the situation is much more complicated. After over a hundred pages of detailed criticism of Zarlino, in the course of which he appears to dismiss both pure just intonation and Ptolemy's scale, Galilei then gives a defence of equal temperament, in order to oblige "some Aristoxenian friends of mine" ("alcuni Aristossenici amici miei")[13]. Finally, this too is rejected, and we read with amazement, and considerable annoyance, that well-trained singers do indeed sing all intervals in just intonation, and that, since their system of intonation is therefore necessarily unstable, it is very complicated and difficult to describe—Galilei would need another whole book to do so[14]. Thus Galilei too evades any attempt at solving the problem of instability, though he at least admits its existence. But what is peculiarly exasperating for the historian is that he now evidently agrees with Zarlino and that the whole controversy has been a bogus one. Moreover, he even explicitly admits this agreement: if Zarlino will abandon his distinction between natural and artificial scales and instruments, Galilei says, "I will at once admit that what we sing today agrees more with this syntonon of Ptolemy than with any other distribution"[15].

[11] *Soppl.*, pp. 140-9.
[12] *Dialogo*, pp. 30-1, 39.
[13] *Discorso*, pp. 109-17.
[14] Ibid., pp. 117-8. On p. 36 Vincenzo announces that he will give this description in a shortly to be published treatise on dissonances. But in his manuscript treatise (v. infra note 18) there are only two mentions of the subject. In the first he says (i. 149v) that we sing Ptolemy's syntonon "con le conditioni però da me avvertite"; in the second (i. 194), that equal temperament differs "pochissimo" from what we sing.
[15] *Discorso*, p. 124: "io subito confesserò che quello che noi hoggi cantiamo convenga più che con altra Distributione con il medesimo Sintono di Tolomeo".

How far Galilei could go in the way of dishonesty is shown by his formal, public declaration, at the beginning of the *Discorso*, that the *Dialogo* is entirely his own work and owes nothing to any contemporary thinker [16]; whereas we know, thanks to Palisca's publication of his correspondence with Mei [17], that large parts of the work are taken, nearly word for word, from the latter's letters, and that it was from Mei that he took the whole general theory of the inferiority of modern polyphonic music, decadent, over-complicated and hedonistic, compared with good, ancient monody, simple and by its marvellous effects leading to moral improvement. Galilei's acceptance of this theory is another cause of confusion and inconsistency. This is particularly striking in the two long manuscript treatises on counterpoint and the use of dissonance [18]. Though he is intensely interested in modern music and very knowledgeable about it—he gives a very competent sketch of its history from the late fifteenth century onwards [19]—he feels obliged to preface the treatises with a sweeping denigration of all polyphony and a eulogy of the good, intellectual, moral music of antiquity [20]; we are then given detailed rules and advice for composing this decadent, immoral modern music. It is evident that he had a keen appreciation of contemporary polyphonic music, especially that of Cipriano de Rore, whom he regularly cites as a model for young composers. Yet, after praising Rore for having been great enough to know when to break rules, as did Michelangelo, Raphael and Donatello, and admiring the particular qualities of various of his madrigals, e.g. the "mestitia . . . con tanto artifizio senz'alcuna affettatione espressa" of "Come

[16] Ibid., p. 14.
[17] Mei, *Letters*, pp. 73-7.
[18] MSS. cit. (supra note 8): (a) Treatise on Counterpoint (no title; begins: "L'arte de la Practica del moderno Contrapunto"); (b) *Discorso intorno all'uso delle Dissonanze*. There are three versions of each treatise. The first version has additions, mostly at the bottom of the page, which are incorporated in the second; this in turn has additions which appear in the text of the third version. The three versions of each, in order of composition, are as follows: (a) ii. 3-54ᵛ, i. 6-51ᵛ, i. 55-103ᵛ; (b) ii. 55-120, i. 104-147ᵛ, i. 148-196ᵛ.
[19] MS. cit. (b), i. 181ᵛ-186ᵛ.
[20] Ibid. (a), i. 57-60; (b), i. 148, 167, 194ᵛ.

havran fin le dolorose tempre" [21], he ends his treatise a few pages later by quoting Sadoleto's Platonic condemnation of polyphonic music as being like the meaningless twittering of birds [22], and expands it, explicitly including his own contemporaries, in exactly the same terms as those of the Mei parts of the *Dialogo* [23].

This crookedness, inconsistency and evasion on the part of Galilei is not only annoying for us, but also unfortunate; for he was an original thinker and a widely experienced musician—he claimed to have collected and entablatured over 14,000 pieces of music [24], and he had some very interesting things to say; but his line of thought is constantly side-tracked and distorted by his obsessive need to contradict Zarlino and to adhere to Mei's contempt for modern music.

One of the points on which Galilei did genuinely, perhaps, disagree with Zarlino was the latter's use of the categories "natural" and "artificial" to support his contention that singers used just intonation. Zarlino's argument runs [25]: nature is superior to art, hence art imitates nature, but nature never imitates art; the human voice is natural, as opposed to man-made musical instruments; just intonation is natural, as opposed to the tempered intonation used by instruments; singers therefore must use just intonation and cannot possibly imitate the equal or meantone temperament of instruments; moreover, they must use the "sintono naturale", pure just intonation, and not the "sintono artifiziale", Ptolemy's scale. Galilei denied both that the distinction between natural voices and artificial instruments was valid or relevant, and that any scale is more natural than another. With regard to the former denial he was in a strong position; the relevant distinction is between free and fixed intonation, and the voice just happens to be freer than any instrument then in use, but

[21] Ibid. (b), i. 190-1; cf. infra Ch. V, p. 78.
[22] Jacobus Sadoletus, *Opera quae extant omnia*, Verona, 1737-8, iii, 111-2, 115-6.
[23] MS. cit. (b), i. 194ᵛ.
[24] Ibid., (a), i. 100.
[25] *Soppl.*, pp. 8, 18-24, 135-40, 143-6.

a one-stringed fiddle would be equally free. With regard to the latter denial, that of the naturalness of scales, he makes some very shrewd points, but typically overstates his case. Since the problems involved here are of considerable importance to musical theory and are still today by no means resolved, his views must be considered in some detail.

"Nature" and "natural" and their opposites are an exceptionally confusing set of terms, as Lovejoy and Boas long ago demonstrated [26], and it will be best to be aware of two groups of meanings that are important in this context. First, "natural" as the opposite of "artificial" can mean the class of everything not man-made, the external physical world, excluding man's soul and its works; in this sense, the complex vibrations of strings are natural, but the arrangement of them by man on a musical instrument into a certain scale is not natural. Second, "natural" as the opposite of "conventional" or "controllable by man" (φύσει as opposed to θέσει) can be used to refer to a datum, a fact we cannot alter, including some facts of human psychology; in this sense, we might claim that just consonances are natural, since all men have always judged them the sweetest, and that tempered consonances are unnatural. After the discovery of overtones there was a strong case for asserting that just intonation was natural in both these senses, i.e. given both in the external world and in human psychology. Before this discovery, just intonation could be natural only in the second sense, as a universal psychological datum, and it was so accepted by the great majority of thinkers of the late sixteenth and the seventeenth century.

Galilei, as we have seen, did eventually accept the superiority of just consonances, and supported his view empirically: the fifth, he says, in the ratio of 3:2 is "more perfect, more sweet than any other form; as I have judged by ear after many, many experiments (since I know of no other better means of achieving

[26] A. O. Lovejoy & G. Boas, *Primitivism and Related Ideas in Antiquity*, Baltimore, 1935.

certainty in this matter)" [27]. And he also eventually agreed with Zarlino that well-trained singers, with "il loro udito perfetto", sing all intervals justly—but he avoided applying the term "natural" to these intervals [28].

With regard to scales, when arguing against Zarlino, he begins by stating that no scales are natural (in the sense of not man-made); the only natural elements in them (in the sense of not arbitrary and conventional) are the ratios of the octave and fifth, "but the latter being divided into four and the former into seven intervals of one or other measure and size is entirely a matter of art" [29].

This is certainly a defensible position, and it is one which fits the multiplicity of scales listed by ancient Greek theorists and which has been confirmed by later knowledge of non-Western scales. But Galilei soon, driven by the need to contradict Zarlino, is led to assert that all scales, and even all intervals, are entirely artificial [30]; which they are in the first sense of "natural", since they are man-made, but not in the second sense, since on his own admission octaves and fifths are in a scale natural data, and, as he later confesses, so are thirds and sixths. He goes on to make the remarkable assertion that all systems of intonation are not only artificial but are consciously learnt from a teacher, and that untaught singers make sounds having little resemblance to those of trained musicians—they are, he says, as different as accidental likenesses of animals in marble or wood are from properly painted animals in a picture [31]. As it stands, this assertion is plainly erroneous: untrained singers do not all sing chaotically out of tune; but it contains an important grain of truth, a truth that was not, I think, recognized by his contemporaries. Keeping Galilei's

[27] *Discorso*, p. 117: "più perfetta, più suave di qual sia altra forma; com'io per il mio udito dopo molte & molte sperienze (poiche con altro mezzo migliore non so potersene haver certezza) ho giudicato".

[28] Ibid., p. 31.

[29] Ibid., p. 21: "ma l'esser divisa questa in quattro & quella in sette intervalli d'una o d'altra misura & grandezza è tutta cosa dell'arte".

[30] Ibid., pp. 77, 80, 86.

[31] Ibid., pp. 98-9.

original proviso of the octave and fifth being unalterable natural data, we may say that it is evident, from the variety of scales that have been and are in use in different cultures, that all systems of intonation are learnt, are passed on from one generation to another, as a language is [32]. It does not of course follow from this, as Galilei implies, that uneducated people cannot speak their native language correctly, but it does follow that systems of intonation are likely to evolve historically. But there remains the important difference between music and verbal language that the former can evolve only within certain natural limits, the octave and the fifth being fixed points, and perhaps, for polyphonic music, as Zarlino and most seventeenth-century theorists maintained, the thirds and sixths.

Or is it possible that custom and tradition can eventually override even these natural limits? The answer is, as we now know, yes, except for the octave; for we have all come to accept equal temperament, in which all the consonances except the octave are slightly false. But the answer was by no means obvious in Galilei's time. He states, as a matter of common experience, that equal temperament, because of its harsh thirds and sixths, is intolerable on keyboard instruments, though bearable in the softer tone of the lute or viol [33]. But with regard to fifths and fourths, which were of course tempered on keyboard instruments as well as on fretted ones, he observes that people have come to prefer narrow fifths and wide fourths, and that a just fifth now sounds harsh and too wide; this, he says quite correctly, is due to their ear being corrupted by constantly hearing tempered fifths [34], though

[32] Galilei (ibid., pp. 81-2) compares intervals in music to words in language; both are wholly artificial—only the sound of the voice is natural.

[33] *Discorso*, pp. 127-8; *Dialogo*, pp. 47-8 (cf. p. 32 on the intolerable effects of Pythagorean tuning); MS. cit., iii. 56, 58 (*Discorso particolare intorno all'Unisono*).

[34] *Dialogo*, p. 55: "con più gusto è universalmente intesa la quinta secondo la misura che gli dà Aristosseno, che dentro la sesquialtera sua prima forma, nè da altro credo veramente ciò avvenga, che dall'haverci il mal uso corrotto il senso: imperoche la Quinta dentro la sesquialtera, non solo pare nell'estrema acutezza che ella può andare, ma più tosto che ell'habbia un poco del duro per non dire (insieme con altri d'udito delicato) dell'aspro, dove nella maniera d'Aristosseno pare, che quella poca scarsità gli dia gratia, & la faccia divenire più secondo il gusto d'hoggi, molle & languida". This, says Galilei, shows the great imperfection of modern music.

he later developed a theory that this too was a natural tendency: "la cortese Natura" had arranged that, while wide fifths are intolerable, narrow ones should be pleasing [35].

At some time between the *Dialogo*, 1581, and the *Discorso*, 1589, Galilei made a discovery, which he hoped would radically undermine Zarlino's numerical theory of consonance. This hope was doomed to failure because, as we now know, Zarlino had arrived by arithmetical methods at the ratios of the overtone series, and at the optimum spacing of the major triad [36], based on that series. But the discovery itself was a genuine one, which had some important repercussions, and, on his own saying, Galilei made it empirically ("con il mezzo dell'esperienza delle cose maestra") [37].

After Pythagoras, as Macrobius [38] and Boethius [39] recount, had discovered the ratios of the perfect consonances by listening to the hammers in the blacksmith's shop [40], he went home and confirmed these ratios by various experiments, the first of which consisted of tying different weights to strings of the same length, and in all of them he found the same simple ratios, octave 2:1, fifth 3:2, fourth 4:3 [41]. Here, says Galilei, the story has gone wrong; the weights would have to be in the ratios of 4:1, 9:4, 16:9, that is to say, the weights are not in simple inverse proportion to string-lengths, but in squared inverse proportion [42]. Up

[35] MS. cit., iii. 38 (*Discorso intorno a diversi pareri che hebbono le tre sette piu famose degli antichi Musici; intorno alla cosa de suoni, et degl'accbordi*); cf. *Dialogo*, p. 47.

[36] *Soppl.*, pp. 100-2. Galilei (MS. cit., i. 84^v) criticizes Zarlino on this point and sneers at his use of the natural notes of the trombone to back up his theory.

[37] *Discorso*, pp. 102-4. In the *Dialogo* (pp. 127, 133) Galilei accepts the Pythagoras story.

[38] *Comm. in Somn. Scip.*, II, i.

[39] *De Institutione Musica*, I, x-xi.

[40] The author of the excellent English translation of Macrobius' commentary, W. H. Stahl (Macrobius, *Commentary on the Dream of Scipio*, New York, 1952, p. 187) very commendably experimented with hammers and anvils, but could produce no musical sound.

[41] Cf., e.g., the frontispiece of Gafori's *Theorica Musice* (1480), showing Pythagoras making these experiments all producing the same ratios, shown by the numbers 4, 6, 8, 9, 12, 16 (reproduced in Wittkower, *Architectural Principles*, p. 108).

[42] *Discorso*, pp. 102-4.

to this point we may assume that Galilei really had done experiments with weights, since his rule is correct. But he then goes on to assert that two pipes will produce an octave if "the length and the void, or let us say the diameter of the lower pipe, is double that of the higher" [13], and a fifth if both the length and the diameter are in the ratio of 3:2, i.e. if the volume of the pipes is in a cubed ratio of 8:1 and 27:8. He is thus able to have a tidy mathematical scheme [14]:

> so that the void (i.e. cubic content) of these pipes corresponds to the cube; the weights suspended from the strings to the plane surface; and the strings simply stretched on the instrument to the line.

Here it is evident that Galilei did not do any experiments, since the pitch of a pipe is a function of its length and not of its cubic capacity. In his unpublished *Discorso intorno alla diversità delle forme del Diapason* [15], he asks the question: what interval would be given by two pipes of the same diameter but one of which is double the length of the other? and answers that it would be an equally tempered major third, which, by his own erroneous rule, is correct. We can see that here the mathematical scheme has been elaborated without even the most rudimentary empirical check, and it has gone wrong.

One of Vincenzo's main motives for doing this was to smash Zarlino's mathematical scheme, which was empirically verified by the monochord [16] and was correct. He is now able to argue that there is no reason to choose the simple ratios 2:1, 3:2 and so on, rather than the squared or cubed ones. This argument, if one omits the cubed ratios, was at that date a strong one, and it is one that his son, the great Galilei, took up and answered. But there was

[13] Ibid., p. 105: "la lunghezza & il vacuo o vogliamo dire il Diametro della grave [canna] sia dupla dell'acuta".

[14] Ibid.: "di maniera che il vacuo de queste [canne] corrisponde al Cubo, i pesi sospesi alle corde, alle superficie, & le corde semplicemente tese nello strumento alla Linea".

[15] Ms. cit., iii. 50ᵛ; Galilei here (iii. 49-51) also claims that 8:1 is the true form of the octave, since all the consonant ratios can be found in the numbers 1 to 8—another rather crazy attempt to smash Zarlino's *senario*.

[16] Cf. Zarlino, *Soppl.*, p. 31, on the monochord.

another curious consequence of Galilei's discovery of the tension law.

When later musical scientists, such as Mersenne [17] and Christiaan Huygens [18], had occasion to mention Pythagoras's discovery of the consonant ratios, they dismissed as just mistaken legend the story that he confirmed these by an experiment with weights; for it was now known, thanks to Galilei, that weights would have given squared, not simple ratios—he must have verified them only by using a monochord. But this is not what Sir Isaac Newton did. As P. M. Rattansi and J. E. McGuire have shown in a brilliant article [19], Newton, like Francis Bacon, took the legends of antiquity very seriously, and believed in a tradition of ancient wisdom, whose deep truths were hidden in a veil of fables. In the 1690's he was working on a set of Scholia for a projected, but never published, second edition of the *Principia*; in these one of his chief concerns was to find anticipations of his own scientific discoveries in the surviving writings of the ancient theologians, the *prisci theologi*, and in the fables of antiquity. Among those which, to his own satisfaction, he found was his own inverse square law of gravitational attraction, and he found it in the version of the Pythagoras story given by Macrobius, who of course links this musical discovery with the Pythagorean doctrine of the harmony of the spheres.

Newton explains at length Galilei's law of tension in the form:

> if two strings equal in thickness are stretched by weights appended, these strings will be in unison when the weights are reciprocally as the squares of the lengths of the strings [50].

He then claims that "this argument is subtile, yet became known to the ancients", and that Pythagoras, by applying it to the heavens, understood "that the weights of the Planets towards the Sun were reciprocally as the squares of their distances from the Sun".

[17] See references given in Mersenne, *Corresp.*, I, 203-4.

[18] *Œuvres complètes*, XIX, 362-3.

[19] "Newton and the 'Pipes of Pan' ", *Notes and Records of the Royal Society of London*, XXI, 1966, 108-43.

[50] Ibid., pp. 115-7.

Newton then briefly, from Macrobius [51] and Pliny [52], sketches the Pythagorean harmony of the spheres: the diatonic scale is applied to the distances between the seven planets, and music is produced by the rubbing together of the solid spheres. Newton concludes that "Pythagoras beneath parables of this sort was hiding his own system and the true harmony of the heavens".

What Newton surely meant was that Pythagoras hid his true law of gravity from the vulgar, not only by the parable of solid spheres producing a scale, but also by publishing his discovery of the musical ratios produced by string-lengths under the disguise of their being simple reciprocals of the ratios produced by weights; the wise, such as Newton, knowing by experiment that the weight ratios are squared reciprocals, would realize that Pythagoras was referring to *musica mundana* and not *musica instrumentalis*, and that he was really promulgating the inverse square law of gravitation.

[51] *Comm. in Somn. Scip.*, II, i-iv.
[52] *Naturalis Historia*, II, xxii.

CHAPTER III

GALILEO GALILEI

I mentioned in the last chapter that the great Galilei took up his father's argument that there was no reason to prefer the simple ratios of string-lengths to the squared ratios of tension. Galilei does not state in his *Discorsi* of 1638 that the question had originally been suggested to him by Vincenzo, but it seems very likely that it was. We know that he studied music with his father as a young man, becoming very proficient on the lute and on keyboard instruments, and when Vincenzo was writing the *Discorso* of 1589 Galilei was in his early twenties and already studying mathematics [1]. It seems therefore most improbable that he did not read his father's treatise [2]. But of course the problem may also have been suggested by some later musical theorist, such as Mersenne [3] or Giambattista Doni [4], with both of whom he was in contact.

Galilei's discussion of the problem occurs in the *Prima giornata* of the *Discorsi* [5], in the section on pendulums. One of the three speakers, Sagredo, remarks that he does not find adequate the reason hitherto adduced by musical theorists for asserting that 2:1, 3:2 and so forth are the "forme naturali" of the consonances; this reason is simply the experimental evidence provided by the lengths of strings of equal tension and weight (or thickness). There are, says, Sagredo, three ways of raising the pitch of a

[1] Galileo Galilei, *Le Opere* (ed. nazionale), Florence, 1890-1909, XIX, 594, 599, 602, 604.
[2] We know that he read and admired Mei's *Discorso sopra la musica antica e moderna* in the same year that it was published (1602); see Galilei, *Opere*, X, 86-7.
[3] Mersenne, *Corresp.*, II, 173-6 (letter of Mersenne to Galilei, February 1629); cf. ibid., I, 194-5.
[4] Galilei, *Opere*, XV, 159, 311-2.
[5] *Discorsi e dimostrazioni matematiche*, Leida, 1638; ed. A. Carugo & L. Geymonat, Turin, 1958; *Opere*, VIII, 138 seq.

string: by shortening it, by stretching it, by making it thinner. By the first, for the octave and fifth, you shorten the string to one half and two thirds; by the second, you increase the tension to four and nine fourths; by the third, you decrease the size to one fourth and four ninths (there by "grossezza" Galilei must mean the area of a section through the string). Why, since the vibrations of a string are too fast to be counted, should we prefer the simple ratios to the squared? [6] From the example of the pendulum, we would expect the shorter string to vibrate faster; but this example would lead us to suppose an inverse squared ratio, i.e. that the strings producing the octave and fifth would vibrate four and nine fourths times as fast as the lower ones. To prove that this is not the case, and that the ratio of frequency of vibration for these two consonances is 2:1 and 3:2, Galilei recounts two experiments.

As far as I know, neither his contemporaries nor modern scholars have commented on these experiments [7], though it is by no means self-evident that they could in fact be carried out with the results that Galilei claimed. It is particularly odd that Mersenne should not discuss them, since he published a French version of Galilei's *Discorsi*, where he mentions them, but without comment [8]. Moreover, Mersenne had long ago accepted this law of the simple inverse proportion of frequency to string-length, first formulated by Benedetti, and rediscovered by Beeckman, and had done corroborative experiments using strings long and slack enough for their vibrations to be counted [9].

The first experiment is as follows [10]. Stand a large glass in a vessel filled with water nearly up to the rim of the glass; make the glass give a musical note by rubbing the rim with your finger;

[6] *Opere*, VIII, 143-4.
[7] Carugo and Geymonat (ed. cit., p. 713) quote from a work of D. Bartoli (*Del suono, de' tremori armonici e dell'udito*, Rome, 1679, p. 140), who failed to make the glass experiment work.
[8] Mersenne, *Les Nouvelles Pensées de Galilei*, Paris, 1639, pp. 95-6.
[9] Mersenne, *Corresp.*, I, 136, II, 231-2; *Harmonie Universelle*, Paris, 1636 (facs. ed., Paris, 1965), Livre III des Mouvemens, pp. 161-2; cf. Palisca, "Scientific Empiricism", p. 135.
[10] Galilei, *Opere*, VIII, 142-4.

you will observe "the waves in the water of exactly equal form" ("le onde nell'acqua con estrema equalità formate"); then, if the note suddenly jumps up an octave, "there will appear other smaller waves, which with infinite precision cut in half the first ones" ("si veggono nascere altre onde più minute, le quali con infinita pullitezza tagliano in mezzo ciascuna di quelle prime"). This is an experiment that both speakers, Sagredo and Salviati, have done several times. It seems to me at least questionable, first, because, if the vibrations of a string are too fast to be clearly seen, so will be the waves in the water [11], and secondly, because I have not yet succeeded in making a sounding glass jump an octave.

The second experiment Galilei hit on by chance [12]. He was scraping a brass plate with an iron chisel in order to remove spots from it, when he noticed that sometimes, when he moved the chisel rapidly, there was a whistling sound, "un sibilo molto gagliardo e chiaro". When this occurred the chisel left on the plate a series of little lines ("virgolette sottili"), parallel and exactly equidistant. The faster he moved the chisel, the higher in pitch the whistle and the closer together the lines [13]. He also felt in the hand holding the chisel a trembling like that felt in the larynx if one voices a word, as opposed to whispering it. He then succeeded in making two chisel-strokes which produced sounds exactly a fifth apart, and he tested them with notes on a harpsichord. He counted the number of spaces between the little lines within the same distance, and found that there were forty-five in one stroke and thirty in the other, "which truly is the form attributed to the fifth" ("e misurando poi gl'intervalli delle virgolette dell'una e dell'altra strisciata, si vedeva, la distanza che conteneva quarantacinque spazii dell'una, contenere trenta dell'altra, quale veramente è la forma che si attribuisce al diapente").

[11] Beeckman notes (letter to Mersenne, October 1629; Mersenne, *Corresp.*, II, 279) that the liquid in the glass appears to boil.
[12] *Opere*, VIII, 144-5.
[13] Ibid., VIII, 145: "strisciando ora con maggiore ed ora con minor velocità, il sibilo riusciva di tuono or più acuto ed or più grave; ed osservai, i segni fatti nel suono più acuto esser più spessi, e quelli del più grave più radi . . .".

Galilei had thus discovered a means of recording musical vibrations exactly, permanently and in a form that enabled one to compare frequencies precisely. Why did no one else use it? Why has no one ever used it? Leaving on one side mechanical problems, such as what made the chisel jump so regularly, we can see that there is a flaw in the experiment, even if we accept all the facts as Galilei recounts them. He counted the spaces of the two strokes within the same distance and found the required ratio of 3:2. But on his own saying he moved the chisel faster when producing the higher note; it therefore traversed this distance in less time than the stroke producing the lower note, and, if the ratio of frequencies was 3:2, should have made less than forty-five lines compared with the slower stroke's thirty. The experiment could only possibly have produced valid results if he had compared the number of lines or spaces made during the same unit of time, not within the same distance.

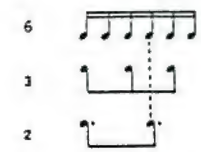
One can only suppose that this was one of Galilei's so-called "thought-experiments", about which he had not thought quite enough, though he tells the story with wonderfully convincing realism. I was greatly relieved to notice this mistake, since otherwise I should have had to waste a lot of time ineffectively scraping brass plates with chisels.

Galilei accepted the third kind of consonance-theory mentioned in Chapter I [14], according to which the degree of consonance of a two-note chord is determined by the rarity or frequency with which the pulses of the sound-waves exactly coincide, e.g. the coincidence occurs for the

octave (2:1): every second pulse of the higher note
fifth (3:2): every third pulse of the higher note
major third (5:4): every fifth pulse of the higher note.

[14] Among other adherents to the theory were: Isaac Beeckman (*Journal*, ed. De Waard, La Haye, 1939, I, 53 seq.; cf. Mersenne, *Corresp.*, I, 606-7, II, 126-7); Descartes (see Mersenne, *Corresp.*, II, 350-1, 370), Mersenne (see H. Ludwig, op. cit., pp. 58-60), and Christiaan Huygens (*Oeuvres*, XX, 34-6).

The first exponent of this theory, Benedetti [15], assumed what Galilei's experiments were meant to prove, namely, that the frequency of sound-waves is in simple inverse proportion to string-length. To see exactly how the pulses coincide or do not, he multiplied the two terms of each ratio together [16], e.g. a fifth, 3:2 = 6



In this same section on pendulums of the *Discorsi* Galilei proposes a visual demonstration of the theory [17]. He believed, wrongly, that the vibrations of pendulums are exactly isochronous [18], and, rightly, that their frequency of vibration is in inverse proportion to the square-root of their length. Let us then take three pendulums that represent a three-note chord of a fifth plus a fourth, thus:

(length of string of pendulum)
16 (= 4^2) will vibrate 2 times while (C)
9 (= 3^2) will vibrate 3 times while (g)
4 (= 2^2) will vibrate 4 times (c)

When we set these in motion we will see

> a beautiful entwining of these strings, with various meetings, but such that at every second vibration of the longest all three arrive unitedly at the same end of the swing [19].

[15] G. B. Benedetti, *Diversarum speculationum mathematicarum & Physicarum Liber*, Taurini, 1585, pp. 277-83; Palisca, "Scientific Empiricism", pp. 104-9.

[16] Benedetti does not explain why he multiplies together the terms of the ratios, but this is the obvious purpose. He does not, as Palisca says, use the products to grade consonances; but he does remark that these products show a wonderful proportion ("non absque mirabili analogia", mistranslated as "logic"), i.e. also contain musical ratios.

[17] *Opere*, VIII, pp. 146 seq.

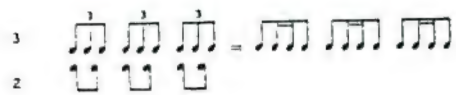
[18] See Galilei, *Discorsi*, ed. Carugo & Geymonat, pp. 694 seq.

[19] Galilei, *Opere*, ibid.: "un intrecciamento vago di essi fili, con incontri varii, ma tale che ad ogni quarta vibrazione del più lungo tutti tre arriveranno al medesimo termine unitamente". Galilei writes "fourth" vibration presumably because he was counting each swing and not each whole vibration.

Thus the eye "can take pleasure in seeing the same games that the ear hears" ("possa recrearsi nel veder i medesimi scherzi che sente l'udito").

Galilei uses his version of the theory not to arrive at the usual grading of consonances, but to explain why the traditionally less perfect consonances may be more pleasing than the octave. The octave, he says, is insipid ("sdolcinata troppo e senza brio") because the pulses, coinciding every other pulse of the higher note, produce a boring rhythm:

whereas a 5th, 3:2, produces:



and this latter rhythm

> makes such a tickling and stimulation of the cartilage of the eardrum that, tempering the sweetness with a dash of sharpness, it seems delightfully to kiss and bite at the same time (fa una titillazione ed un sollecito tale sopra la cartilagine del timpano, che temperando la dolcezza con uno sprezzo d'acrimonia, par che insieme soavemente baci e morda).

Galilei takes the theory to its logical conclusion, a conclusion which disproves it, though oddly enough he did not see this. If consonances are sweet because the pulses of the two notes coincide fairly often, and the less often the less sweet the chord, then the most dissonant intervals of all will be made by two notes whose frequencies are incommensurable [20]. He gives the example of the ratio $\sqrt{2}:1$, which would be an equally tempered augmented fourth (e.g. C-F♯ on a piano); he does not mention temperament, but says that this interval is "similar to a tritone". He repeats this again when using the pendulum analogy, casually making the alarming, but true statement that, if the periods of the two pendulums are incommensurable (e.g. the string-length of one exactly twice that of the other), they will *never* come together again; as the eye will be dazed by these confused movements, so the ear will receive with displeasure the disordered, unregulated tremors of the air.

[20] Beeckman (Mersenne, *Corresp.*, II, 452) noted that with intervals of irrational ratio the pulses would never coincide and that these would therefore be unpleasant to the ear.

This extension of the theory disproves it because it is obvious that, e.g., an equally tempered fifth ($2^{7/12}:1$) is not harsher than, e.g., a just semitone (16:15), although with the former the pulses, after the first, will never coincide again, whereas with the latter they will coincide every fifteenth pulse of the lower note. It is surprising that Galilei should not have realized this, since he was a musician himself and the son of a professional lutenist who was particularly interested in equal temperament.

CHAPTER IV

KEPLER'S CELESTIAL MUSIC

In the long tradition of the music of the spheres [1] Kepler's celestial harmonies [1a] are unique in several respects. First, they are real but soundless, whereas the Greek and mediaeval music of the spheres is either metaphorical, becoming eventually a purely literary theme, or is audible music, which, for various reasons, only very exceptional people, like Pythagoras [2], can hear. Secondly, they are polyphonic (that is, are harmonies in the modern sense of the word), whereas earlier ones, from Plato to Zarlino [3], consist only of scales. Thirdly, they are in just intonation, having consonant thirds and sixths, whereas all earlier systems use Pythagorean intonation, in which the smallest consonance is the fourth. Fourthly, these consonances are geometrically determined, by the regular polygons inscribable in a circle, whereas earlier theorists derive musical intervals arithmetically from simple numerical ratios. Finally, Kepler's *musica mundana* is centred on, and perceived from, the sun. These peculiarities, all of which are interconnected, are the subject of this chapter.

First, there is the basic problem of the objective validity of Kepler's celestial harmonies. Kepler, when finding the ratios of musical consonances in the extreme angular speeds of the planets as seen from the sun [4], allowed himself some margin of error. This was of course quite in accordance with his metaphysics:

[1] There is quite a full bibliography in the article "Harmonie" in *MGG*; cf. also J. Hutton, art. cit. supra Ch. I, p. 1, n. 1.

[1a] For Kepler's musical theory in general see Michael Dickreiter, *Der Musiktheoretiker Johannes Kepler*, Bern & Munich, 1973.

[2] See Walker, *Magic*, p. 37, and J. Hollander, op. cit., p. 29.

[3] Zarlino, in the *Ist. Harm.*, Pt. I, c. vi (*Tutte l'Opere*, I, Venice, 1589, pp. 16-21, cf. pp. 123-6), gives a good account of ancient opinions on the subject.

[4] It is the extreme speeds that give the basic "scales" of each planet; other speeds within these limits are also used to make the harmonies; cf. infra p. 59.

one would not expect to find an exact copy of a geometrical archetype in the natural world. But it is evident that, given a wide enough margin of error, one could find musical ratios in any old set of numbers. Was Kepler, as Athanasius Kircher suggested in his *Musurgia* (1650)[5], playing a game with such lax rules that he was bound to win? Or did he in fact discover a pattern, a regularity which really does exist? I think the answer is that he was not playing too easy a game and that he had every reason to suppose that he had made a genuine discovery—for the following reason. He had been trying, ever since the *Mysterium Cosmographicum* (1596), to find these ratios in the heavens, in the distances between the orbits of the planets, and then in their orbital speeds [6], and he did not find them. It is clear, therefore, that he was not willing to stretch his margin of error so that he could find them wherever he looked for them. He found them only when he placed himself in the sun and looked at the angular speeds of the planets from there.

Kepler's insistence that his celestial harmonies should be real, should be empirically confirmed by astronomical observation, is typical of all his thinking, and in particular of his thinking about music. While searching for, and discovering, purely metaphysical or aesthetic causes for things being as they are and not otherwise—beautiful, simple patterns, mathematically determined and logically interconnected, he always gave absolute priority to empirical evidence [7]; if the theoretical pattern, however beautiful, did not fit the facts, it was discarded. Though Kepler was resolved that his celestial music should not be merely analogical, he by no means despised analogies; and, as we shall see, it is not always clear whether his musical and geometric analogies

[5] Kircher, *Musurgia Universalis*, II, Rome, 1650, p. 379: "Ludere autem in sola proportione, nullius ingenij est, cùm vix ullę numeris subiectae res sint, quae non aliquas ex musicis proportionibus denominationes habeant"; he has just (ibid., pp. 377-8) criticized the largeness of Kepler's margin of error.

[6] See Kepler, *Harmonices Mundi Libri V*, 1619, Lib. V, c. iv (*Gesammelte Werke*, ed. Max Caspar, VI, Munich, 1940, pp. 306-12), and Max Caspar's *Nachbericht* (ibid., pp. 470 seq.).

[7] Cf. E. A. Burtt, *The Metaphysical Foundations of Modern Physical Science*, London, 1949, pp. 50-1.

are only metaphorical or whether they express a real connexion between the two terms of the analogy.

The two novelties in Kepler's celestial music of polyphony and just intonation are closely connected; if thirds and sixths are not admitted as consonances, there can be no polyphony. Although in the sixteenth and seventeenth centuries the question was debatable, Kepler believed, with the majority of competent scholars, that ancient music, though perhaps not strictly monodic, was not polyphonic in any way resembling modern music [8], and that this difference was reflected in the prevailing system of intonation: Pythagorean (in which the thirds and sixths are dissonant) for the ancients, and just (in which they are consonant) for the moderns [9].

For music which is monodic, or in which the interest is concentrated on melody, Pythagorean intonation is more suitable than just, since all the fifths and fourths can be untempered, and the very narrow semitones give greater sharpness to the melody [10]. For polyphonic music such as that of the late sixteenth to the nineteenth centuries, in which the major triad occupies a dominat-

[8] See Walker, "Musical Humanism", sections ix-xi.

[9] Kepler does not explicitly state this connexion in the *Harm. Mundi*, but I think only because it is so obvious; he is a very elliptical writer. It is a commonplace in the 16th-century musical treatises he used; Caspar (Kepler, *Ges. Werke*, VI, 477) gives a list of these, which does not include Zarlino, but Kepler cites him in the *Harm. Mundi* (Kepler, ibid., p. 139). In a letter of 1599 to Herwart von Hohenburg (Kepler, ibid., XIV, 72) Kepler does make this connexion quite clearly; after explaining how the Greeks, owing to their Pythagorean intonation, failed to use the imperfect consonances, he goes on: "Quae cum ita sint, vehementer miror Ursum (et antea quoque mirabar, quam haec scirem), qui veterum Musicam putat longè nobiliorem fuisse nostrâ. Credo gratiam habuisse suam, vocis humanae unius accommodationem ad lyram, quae hodie voluptatis causa passim revocatur: sed unius simplicis vocis modulationem suaviorem esse quatuor vocibus in varietate identitatem tuentibus, numquam credidero. At nuspiam legimus cecinisse illos diversis vocibus in unum". I have not been able to find anything about music in the published works of Reimarus Ursus.

[10] Present-day violinists who believe that they are playing in "natural" or "true" intonation, as opposed to equal temperament, make very narrow semitones by sharpening upward leading-notes and flattening downward ones. In consequence, e.g. G sharp followed by A is sharper than A flat followed by G; and as a result of this their double-stopped thirds and sixths are very harsh. In other words they are attempting to play in Pythagorean intonation. They are also of course pushed towards this kind of intonation by the fact that their instrument is tuned in fifths (cf. J. M. Barbour, op. cit., p. 200).

ing and central position, just intonation has the advantage of making this chord as sweet as possible and in general of making all chords, both major and minor, more consonant, though it has the disadvantages of much greater instability of pitch, of unequal tones, and of much wider semitones (16:15 as compared with 256:243, differing by a comma 81:80).

These remarks are borne out by the history of Western music. Music in the ancient world was monodic, or at least dominated by melody, and the standard intonation was Pythagorean; though Ptolemy, and before him Didymus, gave the ratios of just intonation, they did not accept thirds and sixths as consonances [11]. Pythagorean was the only system known to mediaeval musical theorists; but with the full development of polyphony in the later Middle Ages theorists begin to accept thirds and sixths as "imperfect consonances", though still giving the Pythagorean ratios [12]. If mediaeval musicians were aiming at Pythagorean intonation, their major triads would be no more consonant than their minor ones; and in fact it is not until the later sixteenth century that harmony begins to be dominated by the major triad, and that major and minor tonality begins to replace the modes. This brings us to the period of Zarlino, the first widely read and influential theorist to advocate just intonation, in which the major triad is much sweeter than the minor. The great growth of instrumental music and the development of harmony towards greater freedom of modulation from the sixteenth to the nineteenth centuries are reflected in the eventual triumph of equal temperament as the ideal intonation. For such music equal temperament has enormous advantages: as compared with just intonation or mean-tone temperament, all its fifths and fourths are very nearly true and its semitones narrower; as compared with Pythagorean its thirds and sixths are less dissonant; as compared with any other system, all keys are equally in tune.

[11] Ptolemy, *Harm.*, I, xv, and *MGG*, articles "Intervall", "Didymos".

[12] *MGG*, art. "Intervall", cols. 1344-5.

We may say then that Kepler was right in accepting a real connexion between the growth of polyphony and the prevalence of just intonation as an ideal, though his reasons for this acceptance are not of course the same as those I have just given. For Kepler just intonation and polyphony had finally prevailed because they were natural, that is, they corresponded to the archetypes in the mind of God, on which the created world was modelled, and which are also in the mind of man, the image of God. Modern music and intonation are thus justified and welcomed by Kepler in two ways: first, empirically, because an unprejudiced observer gifted with a good ear can realize that thirds and sixths are consonant—they will please and satisfy him because they correspond to the archetypes in his mind, and, by using a monochord, he can discover that their ratios are 5:4, 6:5, etc.; secondly, the investigator of nature can find these consonances in God's creation, in Kepler's case in the harmony of the spheres, and thus confirm his own empirical knowledge and the instinctively natural, polyphonic practice of modern musicians. The final step is to show by reasons drawn from geometry, that supreme set of archetypes which is coeternal with God, why these ratios and no others produce musical consonances.

Kepler is always most emphatic in affirming that polyphony is a modern invention and therefore quite unknown to the ancients, though for historical evidence he merely refers the reader to Galilei's *Dialogo della Musica antica et moderna* (1581) [13]; and, unlike most of his contemporaries, he sees this as the extraordinary and unique advance that it was, an advance that for him is

[13] In the *Harm. Mundi* Kepler does not mention that Galilei, in the *Dialogo*, was writing against modern polyphony and in favour of a revival of ancient monody and Pythagorean intonation. But in a letter of 1618 to Matthäus Wacker von Wackenfels, Kepler tells how in October 1617, when setting off from Linz to Regensburg, he foresaw a slow journey and therefore took with him Galilei's dialogue, which, though he found the Italian difficult, he read with the greatest pleasure; in it he found valuable information about the ancients, and, although he often disagreed with the author's opinions, he enjoyed the virtuosity with which Galilei expounded views opposite to his own, by extolling ancient music and denigrating modern (Kepler, *Ges. Werke*. XVII, 254). Kepler cites Galilei much more often than any other modern writer on music.

paralleled by the new astronomy and his own discovery of the celestial polyphony. In the Fifth Book of the *Harmonice Mundi*, after he has gone through the "scales" played by each planet, which are like "simple song or monody, the only kind known to the ancients" [14], he begins his chapter on the chords made by all six planets [15], and by five and by four of them, thus [16]:

> Now, Urania, a more majestic sound is needed, while through the harmonic ladder of celestial movements I ascend yet higher, where the true Archetype of the world's structure lies hidden. Follow me, modern musicians, and express your opinion on this matter by means of your arts [17], unknown to antiquity; Nature, always generous with her gifts, has at last, having carried you two thousand years in her womb, brought you forth in these last centuries, you, the first true likenesses of the universe; by your symphonies of various voices, and whispering through your ears, she has revealed her very self, as she exists in her deepest recesses, to the Mind of man, the most beloved daughter of God the Creator.

Though modern music reveals the archetypical structures of the heavens, it is not an imitation of the celestial music, nor derived

[14] Kepler, *Ges. Werke*, VI, 316: "quae proportio est Cantus simplicis seu Monodiae, quam Choralem Musicam dicimus, et quae sola Veteribus fuit cognita, ad cantum plurium vocum, Figuratum dictum, inventum proximorum saeculorum: eadem est proportio Harmoniarum, quas singuli designant Planetae, ad Harmonias junctorum".

[15] The moon is excluded because "Luna seorsim suam Monodiam cantillat, Terris ut cunis assidens" (Kepler, ibid., VI, 323). In the translation of the Fifth Book of the *Harm. Mundi* by Charles Glenn Wallis (in Ptolemy, *The Almagest* [Great Books of the Western World, No. 16], 1952, p. 1040), the last part of this sentence is rendered: "like a dog sitting on the earth", *canis* presumably being read for *cunis*—not a happy emendation; the translation as a whole is very poor. The German translation by Caspar of the whole *Harm. Mundi* (Kepler, *Welt-Harmonik*, Munich-Berlin, 1939) is of course excellent.

[16] Kepler, ibid., VI, 323: "Nunc opus, Uranie, sonitu majore: dum per scalam Harmonicam coelestium motuum, ad altiora conscendo; quà genuinus Archetypus fabricae Mundanae reconditus asservatur. Sequimini Musici moderni, remque vestris artibus, antiquitati non cognitis, censete: vos his saeculis ultimis, prima universitatis exempla genuina, bis millium annorum incubatu, tandem produxit sui nunquam non prodiga Natura: vestris illa vocum variatum concentibus, perque vestras aures, sese ipsam, qualis existat penitissimo sinu, Menti humanae, Dei Creatoris filiae dilectissimae insusurravit".

[17] I.e. "prove me right by using justly intoned polyphony". In a side-note Kepler suggests that modern composers should write six-part motets on one of the Psalms, or some other scriptural text, in return for this eulogy he has given them. Kepler will see that they are published, and says that "Qui propius Musicam coelestem exprimet hoc opere descriptam; huic Clio sertum, Urania Venerem sponsam spondent".

from it; but both are likenesses of the same archetypes, the geometric beauties coeternal with the Creator; and modern music, as we shall see, thereby even allows us to experience something of God's satisfaction in His own handiwork.

Kepler's whole-hearted and joyous acceptance of polyphony as a step forward, comparable in importance with the Copernican revolution, is in marked contrast to the attitude of his contemporaries, even of those who also believed that ancient music was monodic. Zarlino and his followers, such as the composers of *musique mesurée à l'antique* or the Florentine Camerata of Bardi, concede that modern music has acquired additional sweetness and variety through the use of polyphony, but they also believe that, with regard to rhythm and the treatment of text, we still have much to learn from the ancients [18]; there is no feeling that music has acquired another dimension, but merely that one aspect of the art has been elaborated, while another equally, or even more important aspect has degenerated [19].

Another, more subtle contrast with Kepler is provided by Sethus Calvisius, with whom Kepler had a long correspondence on music and on chronology [20], and whose musical treatises he recommends, rather lukewarmly, in the *Harmonice Mundi* [21]. Calvisius, in his essay *De Initio et Progressu Musices, aliisque rebus eo spectantibus* (1600) [22], gives a competent, if brief history of musical theory and practice from the Flood to the present day, and from it Kepler could have gathered all the elements necessary to produce a realization of musical progress. The ancients rejected thirds and sixths; their music was monodic or nearly so; at some

[18] See Walker, "Musical Humanism", section ix.

[19] G. M. Artusi, another author whom Kepler cites (*Ges. Werke*, VI, 181, 182, 185) is almost as severe on modern polyphony as Galilei and Mei. In his *L'Artusi overo delle Imperfettioni della Moderna Musica Ragionamenti due*, Venice, 1600, polyphony is condemned as positively pernicious, because, by its mixture and confusion of rhythms, modes and genera, it prevents the production of the "effects".

[20] The letters concerning music are: Kepler, *Ges. Werke*, XV, 469 seq.; XVI, 47 seq., 55 seq., 216 seq.; XVIII, 455 seq.

[21] Kepler, ibid., VI, 185.

[22] Calvisius, *Exercitationes Musicae Duae. Quarum Prior est, de Modis musicis, quos vulgò Tonos vocant, rectè cognoscendis, & dijudicandis. Posterior, de Initio* . . . . , Lipsiae, 1600.

time in the Middle Ages polyphony was invented, and soon became decadently over-complicated [23]; at the time of the Reformation, "the repurging of celestial doctrine, together with other good arts and languages", an improvement in musical style began, especially in the treatment of text, which has reached its culmination with Orlando di Lasso and other more recent composers [24]. Music has now attained such heights that no further progress seems possible; all we can do now is to use it to thank God that in this last age of the world He has advanced this art, "among the other liberal arts, to its highest perfection", as a prelude to the music of the Church Triumphant in heaven, soon to begin and never to cease [25]. Unlike Kepler, Calvisius however, though he knows and states that the ancients had no polyphony, never singles out this fact as an example of progress, and he sees the present good state of music on a par with that of the other liberal arts, which have been revived after the long mediaeval darkness.

The main reason, according to Kepler, why this musical revelation and revolution was so long delayed was that the ancients did not stay close enough to empirically established facts, to the judgement of the ear. In the preface to the Third Book of the *Harmonice Mundi*, which deals with practical music, Kepler gives a brief history of intonation. The Pythagoreans discovered by ear the perfect consonances of the octave, fifth and fourth, and their ratios (2:1, 3:2, 4:3); but then turned away too soon from the evidence of their ears and towards speculation in numbers—a double error, first in that musical theory must not only start from observation but also be constantly checked by it, and

[23] Calvisius, op. cit., pp. 91-4, 124-8. Calvisius is referring mainly to the complexities of mediaeval rhythmical notation.

[24] Ibid., pp. 133-5: "usque ad coelestis doctrinae, una cum bonis artibus & linguis, repurgationem . . .".

[25] Ibid., p. 138 (last page): "quod hoc ultimo mundi articulo, inter alias liberales artes, hanc etiam ad summam perfectionem deducere, & quasi προαύλιον praelusionem fieri voluit [sc. Deus], perfectissimae illius Musicae in vita coelesti, ab universo triumphantis Ecclesiae & beatorum Angelorum choro, propediem inchoandae, & per omnem aeternitatem continuandae".

secondly, in that the grounds of consonance must be sought not in numbers, but in geometry [26]:

> The Pythagoreans were so addicted to this kind of philosophizing in numbers, that they failed to keep to the judgment of their ears, though it was by means of this that they had initially been brought to this philosophy; they defined solely by their numbers what is a melodic interval and what is not, what is consonant and what dissonant, thus doing violence to the natural instinctive judgment of the ear.

Thus misled, the Pythagoreans, and following them Plato [27], restricted consonances to ratios made out of their tetractys (1, 2, 3, 4), and therefore failed to include thirds and sixths, without which there can be no polyphony. They wrongly accepted as a melodic interval the Pythagorean semitone, or Platonic Limma, 256:243 (the difference between two major tones and a fourth), and wrongly excluded the minor tone, 10:9 (the difference between a major tone and a just major third). This "harmonic tyranny" continued until the time of Ptolemy, who, maintaining the judgment of the ear against Pythagorean philosophy, admitted as melodic intervals the minor tone and just semitone, and gave the ratios of just thirds and sixths. But Ptolemy, though he had thus emended the Pythagoreans' system and rightly trusted his ear, was still misled by their preoccupation with "abstract numbers", and in consequence both wrongly excluded thirds and sixths from the consonances, which "all well-eared musicians of today" accept, and wrongly included among the melodic intervals a division of the fourth into 7:6 and 8:7, which is "most abhorrent to the ears of all men" [28].

[26] Kepler, *Ges. Werke*, VI, 99: "Huic enim philosophandi formae per Numeros, tantopere fuerunt dediti Pythagoraei; ut jam ne aurium quidem judicio starent, quarum tamen indicijs ad Philosophiam hanc initio perventum erat: sed quid concinnum esset, quid inconcinnum; quid consonum, quid dissonum, ex solis Numeris definirent, vim facientes instinctui naturali auditus".

[27] Kepler, ibid., VI, 94-5, 100; cf. his earlier criticism of Plato in a letter of 1599, ibid., XIV, 71-2.

[28] Kepler, ibid., VI, 99; Ptolemy, *Harm.*, I, xv. The appendix to the *Harm. Mundi* contains a critique of Ptolemy's musical analogies (Kepler, ibid., VI, 369 seq.).

Kepler had several reasons for insisting that the causes of consonance must be sought not in numbers but in geometrical figures. First, one cannot find any sufficient reason why God should have chosen the numbers 1, 2, 3, 4, 5, 6, as those out of which consonances should be generated, and have excluded 7, 11, 13, etc.[29]. The reason given in the *Timaeus*, namely, the two families of squares and cubes generated by the triad 1, 2, 3, itself the principle of all things:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & & & 9 & \\ 8 & & & & & & 27 \end{array}$$

is no good, because it excludes the number 5, "which will not allow itself to be robbed of its right of citizenship among the sources of consonances" [30], that is, of the thirds and sixths, all of which in their ratios have the number 5. Secondly, numbers are not suitable as causes of musical intervals, because the terms of musical ratios are continuous, not discrete quantities, and therefore these causes must be sought in geometrical figures [31]. Finally, numbers are metaphysically and epistemologically inferior to geometrical figures and proportions. Numbers do not exist in physical things, but only "dispersed units" so exist; numbers are thus abstract, in the sense that an Aristotelian *tabula rasa* mind could develop them by abstraction from the repetitive sense-experience of any kind of unit—they are "of second, even of third or fourth intention" [32]. But this is not true of geometrical figures

[29] Kepler, ibid., VI, 100; cf. infra p. 49 on Kepler's rejection of harmonic proportion.

[30] Ibid.: "Nam causa illa de Ternario principiorum, et familiâ quadratorum et cuborum inde deductâ, causa est nulla; cùm quinarius ab illa exulet, qui sibi inter Musicorum intervallorum Ortum jus civitatis eripi non patitur"; cf. ibid., VI, 94-5; Plato, *Timaeus*, 35 B.

[31] Kepler, ibid.: "Cùm enim intervallorum Consonorum termini, sint quantitates continuae: causas quoque quae illa segregant à Dissonis, oportet ex familia peti continuarum quantitatum, non ex Numeris abstractis, ut quantitate discretâ . . .".

[32] Kepler, ibid., VI, 431 (*Apologia* against Fludd): "Omnis numerus, ut sit numerus, menti inesse debet, ut docet Aristoteles; in sensibus inque materia numerus non est, sed unitates dispersae"; p. 212: "Numerus definitur esse multitudo ex

and proportions; these do exist, as imperfect copies, in physical things; and the mind or soul recognizes and classes them by comparing them with the God-implanted archetypes within itself [33]. In the *Harmonice Mundi* Kepler quotes a long passage from Proclus's introduction to his commentary on the First Book of Euclid, which is a defence of the Platonic doctrine, that all mathematical ideas exist innate in the soul, against the Aristotelian epistemology of their being universals abstracted from multiple sense-experience. He then concedes that Aristotle was right as far as numbers are concerned, and was right to refute Pythagorean number-philosophy, and Kepler himself rejects Plato's numerology in the *Republic*; but "with regard to continuous quantities I am entirely in agreement with Proclus" [34].

This inferiority of numbers to geometric figures and ratios is important for Kepler's attitude to analogies or symbols. Analogies based purely on numbers correspond to no archetype in the soul of man or mind of God, whereas geometric analogies do so correspond, and, in many cases, are therefore more than analogies: they display the reasons why God created things as they are and not otherwise, or why we are pleased or displeased with certain experiences. Not only in the *Apologia* against Fludd, but elsewhere in the *Harmonice Mundi*, Kepler takes care explicitly to reject any number-symbols which might suggest themselves to the reader; for example, there are six possible consonant triads in the diatonic scale—this fact is *not* to be explained by the six days of creation and the Trinity [35].

Harmony, musical or of any other kind, consists in the mind's recognizing and classing certain proportions between two or

unitatibus conflata . . ."; p. 222: "sunt enim illi [sc. numeri] secundae quodammodo intentionis, imò et tertiae, et quartae, et cujus non est dicere terminum: nec habent in se quicquam, quod non vel à quantitatibus, vel ab alijs veris et realibus entibus, vel etiam à varijs Mentis intentionibus acceperint".

[33] Kepler, ibid., VI, 215-6, the mind recognizes these proportions intellectually, the soul instinctively.

[34] Ibid., VI, 218-22: "De numeris quidem haud contenderim; quin Aristoteles rectè refutaverit Pythagoricos . . . At quod attinet quantitates continuas, omninò adsentior Proclo".

[35] Ibid., VI, 123.

more continuous quantities by means of comparing them with archetypical geometric figures. Now we know by experience that there are seven musical consonances, which have these ratios 2:1, 3:2, 4:3, 5:4, 6:5, 5:3, 8:5, and which can be multiplied indefinitely by doubling their ratios, i.e. by inserting octaves (e.g. 3:2, a fifth, when doubled becomes 3:1, a twelfth). What class of geometric figures will yield these ratios and no others? As early as 1599 Kepler was looking for the answer in the arcs of a circle cut off by regular, geometrically constructable, inscribed polygons [36].

There are two main reasons why he should have looked here. First, he had already had at least partial success in using the five regular Platonic solids to account for the number of planets and the size of their orbits [37]; and in the *Mysterium Cosmographicum* he had, as he later wrote [38],

> wrongly attempted to deduce the number and ratios [of the consonances] from the five regular solid bodies, whereas the truth is rather that both the five regular solid figures and the musical harmonies and divisions of the monochord have a common origin in the regular plane figures,

that is, the number of the regular solids is determined by their surfaces, which must be regular polygons, and the three basic regular polygons, equilateral triangle, square and pentagon, can generate only five solids. Secondly, Kepler had archetypical reasons for using the divisions of a circle rather than of any other figure. One of his favourite analogies, which is certainly more than a metaphor, is that of the sphere representing the Trinity:

[36] Ibid., XIII, 349-50, letter to Herwart von Hohenburg, dated 30 May 1599 (consonances from arcs of a circle); XIV, 29-37, letter to same, 6 August 1599 (consonances from regular inscribed figures).

[37] See A. Koyré, *La Révolution astronomique*, Paris, 1961, pp. 143 seq.

[38] Kepler, ibid., VI, 119: "Legat curiosus lector, quae de his sectionibus ante annos 22 scripsi in *Mysterio Cosmographico*, Capite XII et perpendat, quomodo fuerim illo loco hallucinatus super causis sectionum et Harmoniarum; perperam nisus earum numerum et rationes deducere ex numero quinque corporum Regularium solidorum: cùm verum sit hoc potius, tam quinque figuras solidas, quàm Harmonias Musicas et chordae sectiones, communem habere originem ex figuris Regularibus planis". *Myst. Cosm.* (1596) on consonances in *Ges. Werke*, I, 40-3.

the centre is the Father, the surface the Son, and the intervening space the Holy Ghost [39]. In the *Harmonice Mundi*, when explaining his use of the circle as the cause of consonances, he recalls this "symbolisatio" and extends it. A section through the centre of the sphere produces the plane figure of a circle, which represents the soul of man; this section is made by rotating a straight line, representing corporeal form, which extends from the centre of the sphere to any point on its surface; thus the soul is to the body as a curve to a straight line, that is, "incommunicable and incommensurable"; and the soul is to God as a circle to a sphere, that is, partaking of the divine three-dimensional sphericity, but joined to, and shaping, the plane generated by the bodily line. "Which cause", continues Kepler, "established the Circle as the subject and source of terms for harmonic proportions" [40].

Using only a rule and compasses, one can divide the circumference of a circle into equal parts in only four basic ways (with one exception, the pentecaidecagon, which will be dealt with later), namely, by inscribing in it its diameter, an equilateral triangle, a square [41], and a pentagon; by continuously doubling the number of sides of these figures an infinite number of further divisions is possible. Figures, such as the heptagon, which cannot be so constructed, are not demonstrable, and are thus "unknowable", even to God [42]; they are therefore excluded from the archetypes. The arcs cut off by these basic demonstrable figures provide the following ratios by comparing the arc subtended by one side with the whole circumference, and the arc subtended by the remaining sides with the whole:

	One side to whole	*Residue to whole*
diameter	1/2:1, octave	1/2:1
triangle	1:3, twelfth	2:3, fifth
square	1:4, double octave	3:4, fourth
pentagon	1:5, double octave plus major third	4:5, major third

[39] Ibid., I, 23-4.
[40] Ibid., VI, 224.
[41] The square is really a doubling of the diameter.
[42] Kepler, ibid., VI, 47 seq.

This gives us all but three of the seven basic consonances: minor third and sixth, and major sixth. The last can be obtained by dividing the pentagon into 2 and 3; which yields 2:5, a tenth, and 3:5, a major sixth. Just as an infinite number of consonances can be generated by doubling the ratios, so there are an infinite number of regular polygons obtainable by doubling the number of sides. By using two of these polygons, hexagon and octagon, we can get the missing consonances: 5:6, minor third, and 5:8, minor sixth [43].

The salient feature of this method of explaining the ratios of consonances is that it does not work very well, and it does not work well because of the thirds and sixths. This is the main point I want to make here: if Kepler had accepted the still current, very ancient Pythagorean and Platonic system of intonation, involving only the ratios 1:2, 2:3, 3:4, he would have had no difficulties at all; but he did not accept it, and that on purely empirical grounds—because, before he set out on his investigation into causes, he had already established by ear that just thirds and sixths are consonant. The Pythagorean system would have fitted Kepler's geometrical explanation so well because he could have defined the admissible polygons as those whose sides are either directly commensurable with the diameter of the circle (the diameter itself) or commensurable in square (the triangle and the square) [44], and he need have used no other figures. We may then take Kepler's word for it that he did originally adopt just intonation solely on the judgment of his ear. He emphatically states this in the *Harmonice Mundi*, and gives as evidence the fact that he already used this system of consonances in the *Mysterium Cosmographicum*, that is, at a time when he was still far from finding any satisfactory theoretical justification for it [45]:

[43] Ibid., VI, 101-18. For the minor third cf. infra p. 53.
[44] (Side of triangle)^2 = 3 (radius)^2; (side of square)^2 = 2 (radius)^2. The hexagon might have given trouble; but I am sure Kepler would have found a way round it.
[45] Kepler, ibid., VI, 119-20: "Igitur vel solo allegato mei Mysterij Cosmographici testimonio, satis est munitus auditus, contra Sophistarum obtrectationes, fidem auribus derogare ausuros circa divisiones adeò minutas, et dijudicationem concordantiarum subtilissimam: quippe cùm videat lector me fidem aurium illo tempore

> The evidence of my book the *Mysterium Cosmographicum* alone will be enough to protect the sense of hearing against the objections of sophists who will dare to deny that the ear can be trusted in such minute divisions [of the monochord] and such very subtle distinctions of consonances. For the reader will see that there I relied on the judgment of the ear in establishing the number of divisions [i.e. consonances], at a time when I was still struggling to find causes, and that I did not then do what the Ancients did. They, having advanced a little way by the judgment of the ear, soon despised their guide and finished the rest of their journey following mistaken Reason, having, as it were, forcibly led their ears astray and ordered them to be deaf.

Moreover it is clear from his correspondence that he was in the habit of using a monochord, and he gives advice on how to achieve more accurate results by checking the consonance one is investigating with its residue; for a major third, e.g., check 4/5 with 1/5, i.e. 1/4, a double octave [46]. He also gives, in the *Harmonice Mundi*, an ingenious method of making audible the slight error in Galilei's approximation to equal temperament [47].

In Kepler's correspondence of the year 1599, when he began his harmonic investigations with the regular polygons, it is always the thirds and sixths that give trouble. The pentagon, necessary for the major third and sixth, is indeed constructable, but its sides are incommensurable with the diameter even in square [48]. The octagon, necessary for the minor third and sixth, also has irrational sides even in square; but in any case by what rule do we allow it to divide the circle into three and five parts, but exclude the division into one and seven, which would produce dissonant intervals? With regard to the pentagon, the answer was

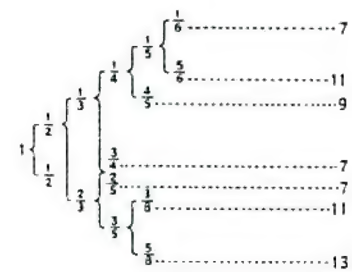
secutum esse, in constituendo sectionum numero, cùm adhuc de causis laborarem; nec idem hic fecisse, quod fecère Veteres; qui aurium judicio progressi aliquatenus, mox contemptis ducibus, reliquum itineris, Rationem erroneam secuti, perfecerunt; auribus vi quasi pertractis, et planè obsurdescere jussis".

[46] Kepler, ibid., XVI, 159; cf. XV, 450.

[47] Ibid., VI, 143-5. Cf. supra p. 16 and infra p. 116.

[48] (Side of pentagon)^2 = (1/2 radius)^2 (10-2$\sqrt{5}$). Kepler, in the First Book of the *Harm. Mundi*, elaborates a detailed system, based on Euclid Book X, for grading the irrationality of the sides of polygons. For trouble about the pentagon in the letters of 1599 see Kepler, ibid., XIV, 30-2, 46-8, 65-6.

to be that its irrationality involves the "divine" proportion of the Golden Section, to which I shall return. For the octagon Kepler tried out various solutions, using regular solids, stars, comparing the arcs not only with the circle but also with the semicircle, etc.[49]; and finally arrived at the rule he uses in the *Harmonice Mundi*, namely, that the harmonic section of a circle must be such that the two parts compared both with the whole and also with each other produce ratios that do not include numbers such as 7, 9, 11, 13, which are the number of sides of undemonstrable figures (heptagon, etc.) [50]. Thus the octagon may divide the circle into three and five parts, since 3:5, 5:8, 3:8 include no "ungeometric" numbers, but not into one and seven parts. By means of this rule, which comes dangerously close to being arithmetic rather than geometric, Kepler already in 1599 gives the neat table of consonant ratios that appears in the *Harmonice Mundi* [51]:



These fractions are generated by adding numerator and denominator to form a new denominator, which has as numerators both the numbers of the previous fraction, the generation being blocked by the appearance of an "ungeometric" number.

These sections of a circle or a string are not of course harmonic in the usual mathematical sense of the term. Kepler does define ordinary harmonic proportion and give the formula for finding an harmonic mean [52] (if a > b > c are in harmonic proportion,

[49] Kepler, ibid., XVI, 31-8, 46-8.

[50] Ibid., XVI, 48, 66.

[51] Ibid., VI, 118.

[52] Ibid., VI, 120-1.

4

$b = \frac{2ac}{a+c}$). But he rejects the use of harmonic proportion, mentioned in Chapter I, to generate the intervals of just intonation. In terms of string-lengths, the harmonic division of an octave, $1:\frac{1}{2}$, gives $1, \frac{2}{3}, \frac{1}{2}$, i.e. a fifth $(1:\frac{2}{3})$ and a fourth) $(\frac{2}{3}:\frac{1}{2} = 1:\frac{3}{4})$. Similarly, the harmonic division of a fifth yields a major and a minor third; and that of a major third yields a major and a minor tone. His grounds for rejecting this theory are that there is an indefinite number of harmonic triads which yield consonant ratios between the extreme terms but not between the mean and the extremes, e.g. $5, \frac{20}{7}, 2$. It was unfortunate that Kepler's dislike of numbers should have led him to reject harmonic proportion, since the harmonic series $1, \frac{1}{2}, \frac{1}{3} \ldots.$ might have led him to the physical basis of consonance, the natural series of overtones [53]. This is the kind of physical explanation that would have delighted him; and, if he had combined it with his regular polygons, he would have had a good reason for excluding the dissonant seventh partial, which, as we shall see, still presented problems for theorists in the mid-eighteenth century [54].

Another difficulty with the regular polygons, which Kepler had cleared up, to his own satisfaction, by 1607, was that raised by the pentecaidecagon. This figure is constructable and cannot be excluded on the grounds that its sides are irrational even in square, since this would entail excluding also the pentagon and octagon. But he was determined to exclude it somehow, and did so on the grounds that it does not have its own independent construction, but can be constructed only by combining a triangle with a pentagon [55]. At first sight it is not clear why Kepler should be so anxious to reject this polygon, since a circle can, on his own principles, be divided by it into twelve and three parts and thus produce consonant ratios. But in the letter to Herwart von Hohenburg of January 1607, which gives a summary of the projected *Harmonice Mundi*, we find the reason, expressed in a typically enigmatic way [56]:

> And so this fifteen-angled figure is sent back among the five foolish virgins. For it comes too late after all the doors have been shut by the numbers 7.9.11.13.

That is to say: to include the pentecaidecagon as a consonance generating polygon would spoil the neatness and elegance of the table of ratios given above.

The acceptance of the pentagon, although its irrationality is greater than that of the triangle or square, is justified, as I have mentioned, by that irrationality involving the Golden Section [57]. This is the proportion between three quantities $(a > b > c)$ which fulfil the two following conditions:

1) that they are in geometric proportion, i.e. that the greatest term is to the middle term as the middle to the least $(\frac{a}{b} = \frac{b}{c}$ or $b^2 = ac)$.

2) That the greatest term is the sum of the two lesser [$a = b + c$; so that the formula is: $\frac{a}{b} = \frac{b}{a\text{-}b}$, or $b^2 = a(a\text{-}b)$; therefore $b = \frac{a}{2}(\sqrt{5}\text{-}1)$].

In other words, a line is divided into two parts in this proportion, if the whole is to the greater part as the greater part is to the less. The side of an inscribed decagon is to the radius of the circle as the greater part to the whole in the Golden Section (decagon side $= \frac{1}{2}$ radius $(\sqrt{5}\text{-}1)$). The square on the side of a pentagon is equal to the square on the side of the decagon inscribed in the same circle plus the square on the radius [(pentagon side)$^2 = r^2 + \frac{r^2}{4}(6\text{-}2\sqrt{5})$; therefore pentagon side $= \frac{r}{2}\sqrt{(10\text{-}2\sqrt{5})}$].

[53] Cf. supra p. 9.
[54] Cf. infra pp. 153-5.
[55] Kepler, ibid., VI, 46-7; cf. XV, 391.

[56] Ibid., XV, 395-6: "Itaque haec figura quindecangulum refertur inter quinque fatuas virgines. Venit enim sero postquam jam januae omnes per numeros 7.9.11.13 occlusae sunt".
[57] Ibid., VI, 42-5, 63-4, 175 seq.

Also the side of a pentagon is to the line joining two of its vertices as the greater part to the whole in the Golden Section. Finally, and most importantly for Kepler, this proportion has the property of generating itself indefinitely: by adding the greater part to the whole one obtains a new whole, and the old whole becomes the new greater part $\left[\dfrac{a+b}{a} = \dfrac{a}{b}\right]$.

Since the pentagon contains the divine proportion within itself, as that between a side and the line joining two vertices, and not only, like the decagon, in relation to the radius of the circle in which it is inscribed [58], Kepler is able to regard the pentagon as the archetypical figure of this proportion and hence of generation in general. In the *Harmonice Mundi* he reinforces the belief that the Golden Section is the archetype of generation by the following consideration [59]. An approximation to this proportion can be obtained by this sequence (Fibonacci numbers):

c or a—b	*b*	*a*
1	1	2
1	2	3
2	3	5
3	5	8
5	8	13
8	13	21 etc.

These sets of numbers satisfy the second of the above two conditions ($c = a-b$); they fail to satisfy the first condition ($ac = b^2$) in such a way that ac alternately exceeds or falls short of b^2 by unity, so that as the sequence is carried on b^2 approaches indefinitely nearer in value to ac or $a(a-b)$:

	$a(a-b)$	b^2
masc.	2	1
fem.	3	4
m.	10	9
f.	24	25
m.	65	64
f.	168	169

[58] Ibid., VI, 63-4.
[59] Ibid., VI, 175.

Where $a(a-b)$ exceeds b^2, the number is, Kepler says, masculine, where it falls short feminine. He then continues [60]:

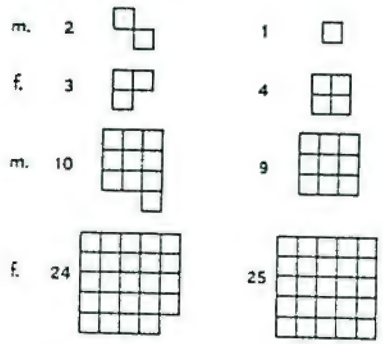
> Since such is the nature of this [golden] section, which is used for the demonstration of the pentagon, and since God the Creator has fitted the laws of generation to that [proportion]—to the genuine and by itself perfect proportion of ineffable terms [has fitted] the propagation of plants which each have their seed within themselves; and to the paired proportions of numbers (of which the one falling short of unity is compensated by the other exceeding [by unity]) [has fitted] the conjunction of male and female—what wonder then, if the progeny of the pentagon, the major third or 4:5 and minor third, 5:6 [61], move our souls, images of God, to emotions comparable to the business of generation.

Kepler is so fond of sexual, male-female, analogies [62] that we become inclined to accept them even when, as in this case, it is not obvious why he has chosen one term for male and the other for female. In a letter of 1608 to Joachim Tanckius we find a fuller treatment of the Golden Section series, though here he does not connect it with major and minor thirds. As in the *Harmonice Mundi*, Kepler represents the numbers thus:

[60] Ibid., VI, 175-6: "Haec cùm sit natura hujus sectionis, quae ad quinquanguli demonstrationem concurrit; cùmque Creator Deus ad illam conformaverit leges generationis; ad genuinam quidem et seipsâ solâ perfectam proportionem ineffabilium terminorum, rationes plantarum seminarias, quae semen suum in semetipsis habere jussae sunt singulae: adjunctas verò binas Numerorum proportiones (quarum unius deficiens unitas alterius excedente compensetur) conjunctionem maris et foeminae: quid mirum igitur, si etiam soboles quinquanguli Tertia dura seu 4.5. et mollis 5.6. moveat animos, Dei imagines, ad affectus, generationis negocio comparandos?".

[61] Kepler continues this passage by arguing that the minor third (6:5) derives primarily, not from the hexagon, but from the decagon, and therefore is of "the class of five-angled figures". He refers the reader back to Ch. iii of this Book (Third). This must be a mistake for Ch. ii, where, on pp. 115-6, one finds the relevant *Propositio* XIII. I do not find Kepler's argument convincing; but I am not sure that I have understood it fully. Where it suits him, Kepler derives the minor third from the hexagon (see next note).

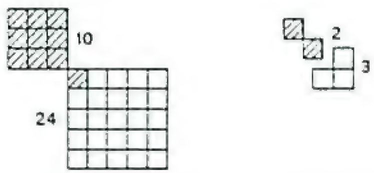
[62] E.g., ibid., VI, 135 (major thirds male, minor female, because former from the irrational pentagon, latter from the rational hexagon), 292 (cube and dodecahedron masculine, octohedron and icosahedron feminine, because latter inscribable in former; tetrahedron androgynous, because inscribable in itself), 326-7 (earth male, Venus female); cf. infra pp. 67-8.



but here he adds the explanation:

> Non puto me posse clarius et palpabilius rem explicare, quam si dicam te videre imagines illic mentulae, hic vulvae.

and moreover the numbers are shown in compromising positions:



This letter[63], as Caspar points out, is important for an understanding of Kepler's attitude to analogies or symbols. He had been sent by Tanckius a work on the monochord by Andreas Reinhard[64], which contained some sexual analogies. Having commented playfully on these, Kepler goes on to say that "by this titillation" Reinhard has excited him to vie with him in finding symbols of male and female; and then, after the long passage on the Golden Section, "which the lecherous feelings roused by Reinhard's speculations had forced out of him"[65], he states[66]:

[63] Ibid., XVI, 154 seq.

[64] *Monochordum*, Leipzig, 1604. I have not been able to see this work, as the British Museum copy has been destroyed.

[65] Kepler, ibid., XVI, 158: "Atque hic excursus esto, quem mihi extorsit prurigo a Reinhardi speculationibus concitata".

[66] Ibid.: "Ludo quippe et ego Symbolis, et opusculum institui, Cabalam Geometricam, quae est de Ideis rerum Naturalium in Geometria: sed ita ludo, ut me ludere non obliviscar. Nihil enim probatur Symbolis, nihil abstrusi eruitur in Naturali

> I too play with symbols, and have planned a little work, Geometric Cabala, which is about the Ideas of natural things in geometry; but I play in such a way that I do not forget that I am playing. For nothing is proved by symbols, nothing hidden is discovered in natural philosophy through geometric symbols; things already known are merely fitted [to them]; unless by sure reasons it can be demonstrated that they are not merely symbolic but are descriptions of the ways in which the two things are connected and of the causes of this connexion.

He gives as an example of a geometric analogy which is also a causal explanation his theory of the weather. Bad weather accompanies certain planetary aspects because there is a Soul of the Earth or *Archeus Subterraneus* which is capable of perceiving geometric relationships, in this case, the angles formed by planetary rays meeting on the earth, and is thereby excited to expel "subterranean humours". Thus "the geometry of the aspects becomes an objective cause"[67], whereas it would be useless to rely on such "symbolisations" as that Saturn brings snow, Mars thunder, Jupiter rain, etc.

When, in the *Harmonice Mundi*, Kepler expounds the triple analogy between the Golden Section, sexual generation, and the emotions aroused by thirds and sixths, is he only "playing and not forgetting that he is playing"? We must be careful not to be misled by his use of the term "play".

In a work that he must have written soon after this letter, since it was published in 1610, the *Tertius Interveniens*[68], Kepler defends astrology against a certain Philipp Feselius, who had recently published a treatise against judiciary astrology. When replying to the argument that, just as the external appearance of plants indicates their medical use, so the colour and light of the planets shows their powers, Feselius had written[69]:

philosophia, per Symbolas geometricas, tantum ante nota accommodantur: nisi certis rationibus evincatur, non tantum esse Symbolica sed esse descriptos connexionis rei utriusque modos et causas".

[67] Ibid.: "geometria aspectuum fit causa objectiva".

[68] Kepler, *Ges. Werke*, IV, 245-6.

[69] Ibid.: "diese *Imagination de signaturis* sey nichts anders dann ein lustige Fantasey müssiger Köpffe / die nit feyren können / und gern etwas zu dichten haben".

> These imagined signatures of things are nothing but a jolly fantasy of idle heads that cannot remain without occupation and like to make up tales.

Kepler's answer to this is quite severe: if by this remark Feselius means to laugh at the doctrine of signatures, his laughter is directed "not only against the most beautiful creatures of God, but also against God Himself". Kepler will interpret this remark in a less blasphemous manner [70]:

> God Himself, since because of His supreme goodness He cannot remain without occupation, has therefore played with the signatures of things, and has represented Himself in the world; and so I sometimes wonder whether the whole of Nature and all the beauty of the heavens is not symbolized in Geometry.

He then recalls his *Mysterium Cosmographicum*, where he had expounded his planetary theory based on the Platonic solids and his analogy between the Trinity and the sphere, and continues:

> Just as God the Creator has played, so he has taught Nature, His image, to play, and indeed to play the same game that He has played before her. (Wie nun Gott der Schöpffer gespielet / also hat er auch die Natur / als sein Ebenbild lehren spielen / und zwar eben das Spiel / das er jhr vorgespielet).

This is why man does not take pleasure in musical intervals which derive from the heptagon or other undemonstrable polygons: "because God has not played with these figures" ("weil Gott mit diesen *figuris* nicht vorgespielt"). And Kepler ends by affirming that, when human reason imitates this divine game, it is not "a silly child's game, but a natural instinct implanted by God" ("So nun Gott und die Natur also vorspielen / so musz dieses der menschlichen Vernunfft nachspielen / kein närrisches Kinderspiel sondern eine von Gott eingepflantzte natürliche anmuhtung seyn. . .").

[70] Ibid.: "dasz Gott selber / da er wegen seiner allerhöchsten güte nicht feyren können / mit den *signaturis rerum* also gespielet / unnd sich selbst in der Welt abgebildet habe: Also dasz es einer ausz meinen Gedancken ist / Ob nicht die gantze Natur und alle himmlische zierligkeit / in der *Geometria symbolisiert* sey".

I think therefore that, even when Kepler seems to be "playing" with analogies, he may be imitating the divine game of the creator, and should perhaps be taken seriously. There is little doubt that, at least by the time of the *Harmonice Mundi*, he believed that the Golden Section analogies showed real causal connexions. Such a deeply pious man would not write in jest that "creator Deus" has fitted the modes of vegetable and animal generation to the archetypical figure of the pentagon. Polyphonic music, with its thirds and sixths, excites and moves us deeply as does sexual intercourse because God has modelled both on the same geometric archetype. There are also, as we have seen and shall see, other archetypical causes of the emotive power of music: the connexions between our music and the celestial harmonies.

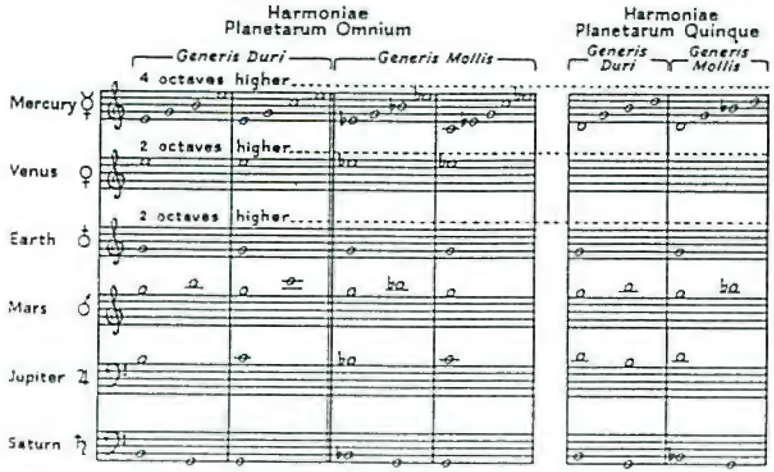
* * *

I shall not here give a general description of Kepler's celestial music, since this has been already done by several modern scholars [71]. I wish merely to discuss a few problems raised by it and aspects of it which, I think, have not been dealt with before.

There are some difficulties in Kepler's planetary chords that are due, at least in part, to his use of the musical terms *durus* and *mollis*. In Book III he describes the two genera, *molle* and *durum*, in such a way that they seem to be the same as our minor and major modes. We are therefore disconcerted when we come to the planetary harmonies in Book V to find that the two chords of all six planets "generis duri" consist of an E minor $^5_3$ and a C major $^6_4$, and those "generis mollis" of an e flat major $^5_3$ and a C minor $^6_4$; the two chords, *durum* and *molle*, for five planets (Venus omitted), on the other hand, are what one would expect:

[71] W. Harburger, *Johannes Keplers kosmische Harmonie*, Leipzig, 1925; A. Koyré; *La Révolution astronomique*, Paris, 1961, pp. 328-45; Caspar's *Nachbericht* to the *Harm. Mundi* (Kepler, *Ges. Werke*, VI, 461 seq.).

a G major $^5_3$ and a G minor $^5_3$:



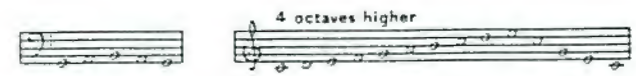
I think that here Kepler is using the terms in their original sense simply to mean respectively any scale or chord which contains a B natural (*durum*) or which contains a B flat (*molle*); this is not surprising, since the theoretical distinction of our major and minor modes was only just beginning to emerge at the time he was writing.

The inclusion of a $^6_4$ chord is odder, since in practice this chord was treated as a dissonance and was banned by most theorists, but not by all, as we shall see in the next chapter. Zarlino gives a defence of the fourth as a consonance, and Kepler had read Zarlino [72]. In his earlier attempts to find harmonies in the orbital speeds of planets Kepler also gives $^6_4$ chords. In one letter he notes that modern musicians may object to the fourth instead of the fifth being at the bottom of the chord, and says that he has answers to this objection; but he does not unfortunately give them [73]. Since he relied so much on the judgment of his ear, he may well just have observed that $^6_4$ chords are nearly as sweet as $^5_3$ ones, and considerably sweeter than $^6_3$ ones.

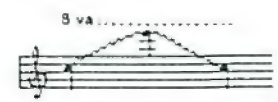
[72] V. infra p. 73, & supra p. 36, n. 9.
[73] Kepler, *Ges. Werke*, XIV, 52; cf. ibid. XIV, 27.

In any case we are not justified in expecting the celestial harmonies to be exactly the same as our music. As I have already said, Kepler makes it clear that our music is not an imitation of celestial music; their relationship is that of two independent products of the same geometric archetypes. Since Kepler does not stress the differences between the two—quite naturally, since he is interested in their similarities—it may make their relationship clearer if I point out two of the most important of these differences.

Each planet has its own scale, determined by its extreme speeds, at aphelion and perihelion [74]. Saturn and Mercury, e.g., have respectively [75]:

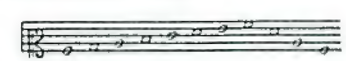


But, as Kepler points out, their passage from the lowest to the highest note and back again is not really articulated into steps of tone and semitone, as in a musical scale, but represents a continuous acceleration and deceleration of the planet's speed, so that, if they actually emitted sounds (which they do not), their scales would sound like a siren giving an air-raid warning. Mercury's scale then would be like:



played with one finger on a violin.

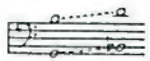
[74] Ibid., VI, 322.
[75] The original has for Mercury's scale:



I am assuming that the C clef should be on the bottom line, as the Frisch edition gives it, and as Koyré transcribes it (*Rév. astr.*, p. 339). This emendation is necessary for Mercury's scale to fit into the planetary chords, and it fits Kepler's planetary speeds better, according to which the highest note of Mercury should be a major sixth and seven octaves above Saturn's lowest note (see Koyré, ibid.). On the other hand, Kepler states that Mercury's scale begins on A (ibid., VI, 321-2), and gives its compass as an octave and a minor third (ibid., VI, 312).

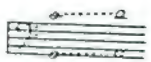
The chords which two or more planets can make are determined by the intervals comprised by their scales and the intervals between these scales [76]. Saturn and Jupiter, e.g. have respectively

and can produce major tenths between

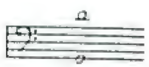


minor tenths between

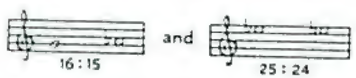
elevenths between



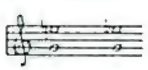
and one twelfth



But with chords of more notes the possibilities become progressively more limited. Since the Earth and Venus have very narrow ranges:



respectively, they can combine with other planets in only a very restricted number of ways. It is these two planets, then, which determine the four possible chords of all six planets, which must all contain the major or minor sixths [77]:



These six-planet chords, already given above, can evidently occur only at very long intervals of time. Kepler doubts whether any of them can yet have happened twice, and conjectures that perhaps it was only at the moment of creation that such a perfectly harmonious combination of all six planets occurred [78]. One

[76] Kepler, ibid., VI, 314-6.

[77] I do not know why Kepler did not use the possibility: A flat and E flat.

[78] Kepler, ibid., VI, 324.

may also, I think, suppose that Kepler believed that at the Last Day the heavens would, before their music ceased for ever, once again sound a perfect chord. In any case, here is another basic difference between earthly and heavenly music. In the latter, the unique piece of divine music begins with a concord, passes through an immense series of dissonances, which are only finally resolved (perhaps) on the final chord; whereas, at least in Kepler's time, human polyphony consists largely of concords, and dissonances are rapidly resolved. Kinds of human music which come near to Kepler's heavenly music would be: the cadenza of a classical concerto, which is a very long interpolation between a $\frac{6}{4}$ chord and its $\frac{5}{3}$ resolution, or a piece written entirely on a pedal-note (e.g. one of Bach's *Musettes*).

When, however, Kepler himself describes the likeness between earthly and heavenly music, he evidently assumes that there will be more than only one or two perfectly consonant chords in the whole course of the world's history [79]:

> The motions of the heavens, therefore, are nothing else but a perennial concert (rational not vocal) tending, through dissonances, through as it were certain suspensions or cadential formulae (by which men imitate those natural dissonances), towards definite and prescribed cadences [80], each chord being of six terms (as of six voices), and by these marks [sc. the cadences] distinguishing and articulating the immensity of time; so that it is no longer a marvel that at last this way of singing in several parts, unknown to the

[79] Ibid., VI, 328: "Nihil igitur sunt motus coelorum, quàm perennis quidam concentus (rationalis non vocalis) per dissonantes tensiones, veluti quasdam Syncopationes vel Cadentias (quibus homines imitantur istas dissonantias naturales) tendens in certas et praescriptas clausulas, singulas sex terminorum (veluti Vocum) ijsque Notis immensitatem Temporis insigniens et distinguens; ut mirum amplius non sit, tandem inventam esse ab Homine, Creatoris sui Simiâ, rationem canendi per concentum, ignotam veteribus; ut scilicet totius Temporis mundani perpetuitatem in brevi aliqua Horae parte, per artificiosam plurium vocum symphoniam luderet, Deique Opificis complacentiam in operibus suis, suavissimo sensu voluptatis, ex hac Dei imitatrice Musicâ perceptae, quadamtenus degustaret".

[80] Kepler, like Calvisius, uses the term *clausula* for cadence. The only time he uses the term *cadentia* (ibid., VI, 182) is when discussing suspensions; he suggests that the word *cadentia* derives from the fact that the dissonant suspended note *falls* to its resolution. In this passage therefore I believe that, in coupling *cadentiae* with suspensions, Kepler was thinking of the regular $\frac{5}{4}$ to $\frac{5}{3}$ suspensions at perfect cadences.

> ancients, should have been invented by Man, the Ape of his Creator; that, namely, he should, by the artificial symphony of several voices, play out, in a brief portion of an hour, the perpetuity of the whole duration of the world, and should to some degree taste of God the Creator's satisfaction in His own works, with a most intensely sweet pleasure gained from this Music that imitates God.

Once again I wish to emphasize that this comparison is not only a metaphor; through the geometric archetypes there is a real causal connexion between the two polyphonies which accounts for their likeness. By these causal analogies between human music and planetary movements, and between music and sexual desire, Kepler gives to music a meaning and value that had not previously been attributed to it, a meaning which only polyphonic music, unknown to the ancients, could possibly have. The marvellous effects of music, emotional, moral and religious, are of course familiar enough; but in that tradition it was always music together with words that produced the "effects", and it was always the words that bore the specific meaning, that determined the particular effect. That music alone could have a precise and profound meaning was, I think, in Kepler's time an entirely novel idea. It is an idea that we have all come to accept, and, although we may find Kepler's explanation of it unconvincing, we cannot claim to have found a better one.

CHAPTER V

THE EXPRESSIVE VALUE OF INTERVALS AND THE PROBLEM OF THE FOURTH

During the second half of the sixteenth century and the first half of the seventeenth several musical theorists dealt with the question: what intervals are suitable for expressing what emotions? Without exception, these authors divide emotions into two large contrasting classes: on the one hand, vigour, energy, joy, but also hardness, harshness, bitterness, and on the other, softness, weakness, sweetness, pity, sadness. The two groups of affective qualities vary somewhat from author to author; sometimes it is harshness that predominates in the former and sweetness in the latter, and sometimes joy in the former and sadness in the latter. But there are always two categories set in opposition one to the other, among which musical intervals, both harmonic and melodic, are distributed. The two qualities that remain constant are vigour in the first group and weakness in the second. For brevity's sake, then, I shall call the first group "vigorous" and the second "weak".

I shall deal first with melodic intervals and then with harmonic ones. The majority of theorists assume two general principles which determine the distribution of melodic intervals into the two classes of emotion: first, the opposition between rising and falling intervals; secondly, the opposition between large and small intervals. Evidently these two principles may conflict. In Mersenne's competition of 1640, as we shall see in the next chapter [1], everyone concerned agreed that small and descending intervals are weak, and that large and ascending ones are vigorous. But in the case of a descending, but eminently vigorous interval, such as the major sixth, it was doubtful whether it was weak be-

[1] V. infra pp. 100-101.

cause it descended or vigorous because it was large. In the case of semitones and minor thirds the principle of size overrode that of direction; these intervals are always weak, whether rising or falling—Mersenne states this explicitly in his *Harmonie Universelle* (1636) [2]:

> Semitones and accidentals represent tears and groans because of their small intervals, which signify weakness; for little intervals, either ascending or descending, are like children, like old people or those who have recently had a long illness, who cannot walk with big steps, and who cover a short space in a long time.

In some theories these two principles are combined in a more subtle and complicated manner. According to Nicola Vicentino, in his *L'Antica Musica ridotta alla moderna Prattica* (1555), the small intervals, semitones and minor thirds are weak when rising and vigorous when falling, whereas the large intervals, tone, major third, fourth and fifth, are very vigorous when rising and very weak when falling. The terms used by Vicentino for the two groups of emotion are: *molle* and *mesto* for the weak, and *incitato* and *allegro* for the vigorous [3]. But in this passage, I think, his line of thought has been distorted by the wish to present a neat and symmetrical scheme of contrasts. For, when dealing elsewhere, separately, with minor and major thirds and tenths, he shows greater musical sense. Discussing the minor third, as both a melodic and a harmonic interval, he states that [4]

> its nature is very feeble, and is rather sad, and likes to descend; it will appear somewhat happy if accompanied with rapid, or very rapid movements; and when it ascends slowly, it will have the nature of a man when he is tired . . . . this consonance is very suitable for sad words when it is prolonged.

[2] Mersenne, *Harm. Univ.*, II des Chants, Pr. xxvi, p. 173: "les demi-tons & dieses representent les pleurs & les gemissemens à raison de leurs petits intervalles qui signifient la foiblesse: car les petits intervalles qui se font en montant ou descendant, sont semblables aux enfans, aux vieillards & à ceux qui reviennent d'une longue maladie, qui ne peuvent cheminer à grand pas, & qui font peu de chemin en beaucoup de temps".

[3] Vicentino, *L'Antica Musica*, Rome, 1555, fols. 19-26.

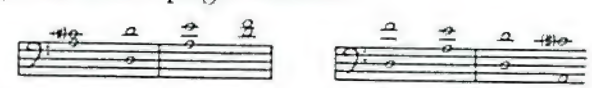
[4] Vicentino, ibid., fol. 33v: "sua natura, laquale è molto debole, et ha del mesto, et volentiera discende; Questa parerà alquanto allegra, quando sarà accompagnata dal moto veloce, & velocissimo; et quando ascenderà con il moto tardo, havrà della natura d'un huomo quando è stracco . . . questa consonanza servirà bene alle parole meste, stando alquando ferma . . .".

Of the major third he says that it is "of a lively and happy nature, and it likes to ascend because of its liveliness" [5]. Likewise the minor tenth is feeble (*debole*) and likes to descend, and the major is very lively (*molto viva*) and likes to ascend [6].

Vincenzo Galilei, in the *Dialogo* of 1581, uses the principle of direction in a similar way. When criticizing the preference in modern polyphony for contrary motion of parts, he reasons thus [7]: it is manifest

> that the [parts] will express the same affect more efficaciously with similar than with contrary [motion]; and that joy and sadness, and the other passions, can be caused in the listener not only by high and low sounds, and by rapid or slow movements, but also by the different quality of the intervals, and even by the same interval when descending or ascending. For the fifth when ascending is sad . . . , and when descending is joyous; and contrariwise the fourth is such when rising, and of the other quality when falling; and the same is observed to happen with the semitone and the other intervals.

It is evident that in this passage Galilei is speaking of melodic intervals, since he mentions semitones, and is giving us a variation of Vicentino's theory. But elsewhere he also takes into account the harmonic progression of the parts. The reason why the fifth and fourth have opposite qualities when rising or falling is that, if these intervals make the bass of the harmony, the descending fifth or the ascending fourth produce a perfect cadence, and the contrary directions a plagal cadence [8]:



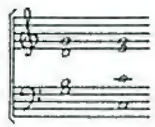
[5] Ibid., fol. 34: "è di natura vivace et allegra, et volontiera ascende per cagione della sua vivacità".

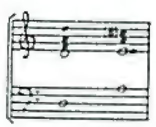
[6] Ibid., fol. 38v.

[7] Galilei, *Dialogo*, p. 76: "che con maggior efficacia son'atte ad esprimere l'istesso affetto con simile, che col diverso; & che l'allegrezza & la mestitia insieme con l'altre passioni, possono esser cagionate nell'uditore non solo con il suono acuto & grave, & col veloce e tardo movimento; ma con la diversa qualità degli intervalli: anzi con l'istesso portato verso il grave, ò verso l'acuto. imperoche la quinta nell'ascendere è mesta, come detto havete, & nel discendere è lieta; & par il contrario la quarta è tale nel salire, & d'altra qualità nel discendere; & l'istesso si vede accadere al Semituono, & alli altri intervalli".

[8] Ibid., p. 75.

The first example, which is in an authentic mode, formed by the harmonic division of the octave, is "of a stable and tranquil nature, without violence, and suitable for inducing in the souls of the listeners grave and severe thoughts, and customs of the same kind". The second, in a plagal mode, formed by the arithmetic division of the octave, is "languid, tearful and timid" [9]. And if for the perfect cadence the mode is that which begins on C, thus:



the effect will be "joyous, excited, and so to speak virile and natural"; and this effect is produced, he says, by the major third and tenth, G-B, C-e [10]. Galilei points out that the rising tone, d-e, of the contralto is very virile ("ha grandemente del virile"), whereas the same interval, even the same notes, in a plagal cadence in the mode on D, would have a "sad and relaxed" effect ("ha del mesto & rimesso"):

And from these examples he draws the conclusion that [11]

> it is truly the bass part that in polyphony gives a song its character.

It is a pity that Galilei did not pursue this line further; for it is only if the harmony were taken into account that all these theories of the expressive value of melodic intervals could have any validity.

Another fruitful line of thought is that suggested by Vicentino's remarks on the tendency of minor thirds to fall and major ones to

[9] Ibid., p. 74: "di natura stabile, & quieto, senza violenza, & atto à indurre negli animi degli uditori pensieri gravi & severi, & costumi da sorti"; "languida, flebile, e timorosa".

[10] Ibid., p. 76.

[11] Ibid.: "che la parte grave sia veramente quella che dà l'aria (nel cantare in consonanza) alla Cantilena".

rise. This idea is developed in a remarkable way by Kepler in the *Harmonice Mundi*. Kepler, as we have seen, for mathematical reasons, classed major thirds as masculine and minor ones as feminine; but he also gives another justification for this classification. The major third is masculine because, when singing e.g. C, D, E, one feels an urge to overcome the semitone and reach the fourth, F. This third, then, is "active and full of efforts" ("actusosa et conatuum plena"); its generative power ("vis γόνιμος") is striving to reach the fourth, and when the semitone is sung and the fourth reached this is like an ejaculation ("quaerens finem suum, scilicet Diatessaron, cujus semitonium est ei [sc. tertiae durae] quasi ἔκχυσις toto conatu quaesita"). The minor third is feminine because, when singing e.g. D, E, F, one feels a tendency to sink back a semitone on to E; this third therefore is passive, and is always sinking to the ground, like a hen ready to be mounted by a cock ("semper se, veluti gallina, sternit humi, promptam insessori gallo") [12].

This explanation of the contrasting character of the two thirds seems to me important and fruitful because it is founded on a conception of the attractive power of the semitone, which produces in the listener the impression that certain melodic lines are likely to rise or to fall—in other terms, the conception of what was later called the leading-note. In a more general way, this conception is important because it introduces into musical theory the psychological notion of expectation, the notion that, when listening to a piece of music in a familiar idiom, one expects, at every moment, that the melody and the harmony will proceed in such or such a manner, an expectation which is often fulfilled and often disappointed [13].

Mersenne, when discussing major and minor thirds and sixths, reproduces this theory of Kepler's, omitting of course the sexual metaphor, in order to explain why the major intervals are "very

[12] Kepler, *Ges. Werke*, VI, 176.

[13] Cf. Leonard B. Meyer, *Emotion and Meaning in Music*, Univ. of Chicago Press, 1956; and cf. Galilei on the tritone (MSS. cit., I, 168v).

suitable for joy, and for expressing virile and courageous actions", and the minor "for flattering and for softening the passions, and for expressing sadness and pain" [14]. He also suggests another reason, which is perhaps valid, as a cause of this difference between the major and minor intervals [15]. The minor sixth, he says, is

> more suitable for expressing great pains and the cries that go with them; the minor third is likewise better for slighter annoyances and for flattering.

They have

> this property by reason of the semitone which represents weakness, because it needs more force to sing the tone.

In fact, a greater increase in the muscular tension of the vocal chords is required to ascend by a greater interval, and, on the other hand, to sing a descending interval produces a feeling of relaxation. This may explain why all the participants of Mersenne's competition agreed that falling intervals were suitable for expressing weakness or death, and rising ones energy and anger.

With regard to harmonic intervals, the attention of all our theorists is concentrated on the thirds and sixths, and one can see, even more clearly than with the melodic intervals, the gradual emergence of our modern modes, major and minor. With regard to their expressive value, the theorists, without exception, attribute the vigorous group to the major and the weak to the minor. Zarlino, when dealing with the imperfect consonances, describes the major ones as "lively and joyful, very sonorous" ("vive & allegre, accompagnate da molta sonorità"), and the minor, he says, "although sweet and soft, incline somewhat towards sadness or langour" ("dolci & soavi, declinano alquanto al mesto,

[14] Mersenne, *Harm. Univ.*, III des Genres, Pr. xviii, p. 188: "fort propres pour la joye, & pour exprimer les actions masles & courageuses"; "pour flater, & pour addoucir les passions, & pour exprimer la tristesse & la douleur".

[15] Ibid.: "plus propre pour exprimer les grandes douleurs avec des cris proportionnez, la Tierce mineure est semblablement meilleure pour exprimer les moindres déplaisirs & pour flater"; "cete propriété à raison du demiton qui represente la foiblesse, parce qu'il faut plus de force pour faire le ton". Cf. I des Cons., Pr. xxxii, p. 80.

overo languido"). On the basis of this contrast, he divides the modes into two classes according to whether there is a major or a minor third on the final: the modes on C, F, G, are "very joyful and lively", whereas those on D, E, A, are rather sad. Since the complete chords, the triads, of the two classes both contain a major and a minor third, Zarlino, like Galilei, explains this contrast by his numerical theory of consonance: the major triads are more vigorous than the minor because they are formed by the harmonic division of the fifth (15, 12, 10), which is more natural than the arithmetic division (6, 5, 4), which produces the minor triad [16].

When he gives advice on the musical expression of the text Zarlino considers both melodic and harmonic intervals. He begins with a rather different version of our two classes of emotion: first, harshness, hardness, cruelty, bitterness ("asprezza, durezza crudeltà, amaritudine"); second, complaint, pain, lamentation, sighs, tears ("pianto, dolore, cordoglio, sospiri, lagrime"). To express the first class one should use melodic intervals "without a semitone, such as the tone and major third", and harmonic intervals such as the major sixth and major thirteenth, "which by their nature are rather harsh" when the lower note is the bass of the chord (i.e. in modern terms, a minor $^{6}_{3}$ chord), and to which one should add suspensions of the fourth or eleventh (i.e. a major $^{6}_{4}$ chord); in general the harmony should be "rather hard and harsh, but of such a kind, however, that it does not offend", and suspensions of the seventh are recommended. For the second class one should use as melodic intervals "the semitone, minor third and suchlike", and as harmonic intervals minor sixths and minor thirteenths on the bass of the chord (i.e. major $^{6}_{3}$ or minor $^{6}_{4}$ chords), "which by their nature are soft and sweet"; in general the harmony should be full of sadness [17].

Later, in the *Annotazioni* of Giambattista Doni, published in 1640, the concentration on thirds and sixths as the principal

[16] Zarlino, *Ist. harm.*, Venice, 1573, III, x, p. 182; cf. M. Shirlaw, op. cit., pp. 44-5.

[17] Ibid., IV, xxxii, pp. 319-21.

sources of affective expression becomes explicit. It is from the imperfect consonances, says Doni, that are derived [18]

> energy and efficacy in moving the effects, because the minor consonances are tearful and sad, and give this character to the chords; and the opposite is true of the major, which are joyful and spirited.

Doni makes this statement because he wishes to prove that the music of the ancients was polyphonic. Without thirds and sixths how could the ancient Greeks have made their songs "as pathetic and efficacious" as the witness of the most weighty authors lead us to believe [19]? The Pythagoreans had banished these consonances because of their superstitious reverence for the tetractys, which stopped the series of consonances at the fourth, and which even made them deaf to the evident sweetness of the major third, to which consonance, says Doni, would well apply what Galileo Galilei had written of the fifth as compared with the octave—that "it kisses and bites us at the same time" [20].

A few years before this work of Doni, Mersenne, in the *Harmonie Universelle*, had reproduced, almost word for word, Zarlino's advice for expressing the two classes of emotions. For him the modes which correspond to our major express "rigour, harshness, bitterness and anger", and those which correspond to the minor "complaints, pain and sighs" [21].

I want now to compare these two groups of emotions, such as we have found them in the theorists of the sixteenth and seventeenth centuries, with the emotions that a modern musician, or even an eighteenth-century one, would attribute to our major and minor modes. I am well aware that this is a very delicate task and that the following comparative scheme is much too simple and

[18] G. B. Doni, *Annotazioni sopra il Compendio de' Generi, e de' Modi della Musica*, Rome, 1640, p. 264: "l'energia, & efficacia in muovere gl'effetti: perche le consonanze minori sono flebili, & mesti; & tali rendono i concenti; & per il contrario le maggiori: che sono allegre, & spiritose".

[19] Ibid., pp. 265-6.

[20] V. supra p. 32.

[21] Mersenne, *Harm. Univ.*, V de la Composition, Pr. x, p. 323v: "la rigueur, l'âpreté, l'amertume & la cholere"; "les plaintes, la douleur, & les soupirs". Cf. Embellissement des Chants, Pr. vii, p. 360.

crude to represent adequately psychological phenomena which are very subtle and variable. But nevertheless this comparison ought to be attempted because there are, I believe, important differences between the groups of the past and those of today, and if we are unaware of these differences we run the risk of wrongly interpreting the intentions of Renaissance composers and the reactions of their audiences. Here then are two lists of emotions; if on these lists one indicates by brackets the emotions that were formerly attributed to the major and minor modes and those that have been attributed to them since the eighteenth century, one finds that the brackets overlap in the middle, but that at both ends there is a symmetrical contrast:

MAJOR

18th–20th centuries: softness, sweetness, joy, vigour
16th–17th centuries: joy, vigour, harshness, bitterness, anger

MINOR

18th–20th centuries: harshness, bitterness, anger, sadness, weakness
16th–17th centuries: sadness, weakness, softness, sweetness

The resemblance to one of Lévi-Strauss's schemes is purely accidental—but I am very conscious that mine is too simple and too symmetrical. To convince himself that I am not wholly wrong with regard to modern reactions to the modes, the reader has only to look through Schubert's *Winterreise* while paying attention to the verbal text and the contrasts between major and minor.

Of all the harmonic intervals only the fourth raised for theorists of our period special difficulties that were almost insurmountable, because it seemed to be both a consonance and a dissonance, two classes which were otherwise exclusive. According to either of the two theories of consonance current at this time [22], the arithmetic theory based on the *senario* and harmonic proportion, or the theory based on the coincidence of vibrations, the fourth was undeniably consonant. Its ratio, 4:3, is superparticular and is contained within the numbers 1 to 6; its vibrations coincide more frequently than those of thirds and sixths. But, since the beginning

[22] V. supra pp. 00-00.

of the sixteenth century, all composers treated the $^{6}_{4}$ chord (the term is anachronistic, but in fact this is the chord in question) as if it were dissonant, that is to say, the note which makes a fourth against the bass must be prepared, be a suspension, and must be resolved downwards by step; the rule concerning resolution remained in force until the nineteenth century. There is a real problem here, to which no satisfactory solution has yet been found. For without doubt this chord is sweeter than a $^{6}_{3}$, which has always been accepted as consonant. According to Helmholtz's theory, based on beats, a major $^{6}_{4}$ is not only more harmonious than a major $^{6}_{3}$ or a minor $^{5}_{3}$, but even than a major $^{5}_{3}$ [23].

The majority of theorists accept, more or less reluctantly, the practical rule which demands that the $^{6}_{4}$ be treated as dissonant, and make desperate attempts to justify this anomaly. Salinas, for example, in his *De Musica* (1577), begins by expounding the reasons in favour of the fourth: the authority of the ancients; the fact that it becomes consonant if it has a fifth under it, whereas no true dissonance can be turned into a consonance in this way; his own experience when he was at Naples, where he often heard the Greeks singing canticles with the fourth in the bass, and the effect was excellent ("& mirabiliter audiendo delectabar"); the example of Josquin des Prés who, in the mass *L'homme armé*, began the two-voice *Et resurrexit* with a fourth; the use of the $^{6}_{4}$ on several instruments, such as the (?) guitar ("psalterium"). But then he justifies the practical rule by this rather feeble argument: since the fourth, resulting from the arithmetic division of the octave, is less perfect than the fifth, resulting from the harmonic division, it likes to be accompanied by the latter; as the vine is to the elm, or woman to man, so is the fourth to the fifth [24]. In another chapter Salinas puts forward a new classification of consonances according to which the fourth would be in the same class as the minor third and the sixths: the perfect consonances are those that can be the final chord in a two-voice piece: octave,

[23] See Shirlaw, op. cit., p. 171.
[24] Francisco Salinas, *De Musica*, Salmanticae, 1577, pp. 55-6.

fifth, major third; the imperfect consonances cannot be: fourth, minor third, the sixths. The perfect consonances can be harmonically divided into two consonances or two melodic intervals; the imperfect cannot be so divided [25].

But there was also a minority of theorists who dared, more or less timidly, to recommend the use of the $^{6}_{4}$ as a consonance. Among this minority one is surprised to find Zarlino. Like Salinas, he invokes the authority of the ancients in favour of the fourth, and the example of Josquin, and he bears independent witness to the use of the $^{6}_{4}$ by the Greeks, whom he has heard in their church at Venice. He believes that perhaps the origin of the modern rule is the doctrine of the Pythagoreans, who accepted as consonant only superparticular and multiple ratios and therefore rejected the eleventh, 8:3 [26]. He gives detailed advice on the use of the fourth, from which one can extract the following principles: (in modern terms) major $^{6}_{3}$ chords and major $^{6}_{4}$ are always good; minor $^{5}_{3}$ and minor $^{6}_{4}$ are less good—they have a "triste effetto". He then firmly states that a major $^{6}_{4}$ is pleasanter to the ear ("piu grata all'Udito") than a major $^{6}_{3}$. His final conclusion is that $^{5}_{3}$ and $^{6}_{3}$ chords are always acceptable, and that the fourth [27]

> can sometimes be used with the third above it [$^{6}_{4}$], especially when the latter is major, although until now all practical musicians have thought little of this [chord]. For, if the fourth accompanied by the major third below it [minor $^{6}_{3}$], which is not really very consonant, is tolerated, I see no reason why [the fourth] accompanied by the major third above it [major $^{6}_{4}$] should not be tolerated, since this [chord] is truly better, as experience will always show us.

The most intrepid defender of the fourth is Andreas Papius, who in 1581 published at Antwerp a long treatise, *De Consonantiis, seu*

[25] Ibid., pp. 69-70.
[26] Zarlino, *Ist.*, Venice, 1589, III, v, pp. 186-8.
[27] Ibid., III, lx, pp. 302-4: "si potrà porre alle volte con la Terza nell'acuto; massimamente quando sarà la maggiore; ancora che questo dall'università dei Musici Prattici fin'hora sia stata poco considerata; percioche se l'accompagnamento della quarta con la Terza maggiore posta nel grave, che non è veramente molto consonante, è sopportata; non sò veder ragione, perche non si dè sopportare l'accompagnamento della Terza maggiore posta nell'acuto; essendo che questo veramente è migliore, come la esperienza ce lo farà sempre vedere".

pro Diatessaron Libri Duo, which is in fact a defence of the $^6_4$ as a consonant chord. Papius is well aware of the originality of his enterprise and of the opposition that it will arouse among professional musicians, although two composers, Jacques de Kerle and Philippe de Monte, had encouraged him to write the book [28]. He welcomes therefore with enthusiasm the support given him by Zarlino's authority. Having resumed the passage I have just quoted, he cries out [29]:

> in this matter he felt the force of truth, and by professing it he has deserved eternal praise.

But unfortunately Zarlino, like the others, was led astray by his wish to justify the practice of modern composers, especially that of his master, Adrian Willaert.

Papius's arguments in favour of the $^6_4$ are on the whole not very convincing. Most of them are based on three principles, all of which are erroneous. The first supposes that the perfect consonances, octave, fifth and fourth, are pleasanter than the imperfect, and that therefore a chord C-g-c or even C-f-c is better than a major triad C-e-g [30]. The second supposes that a chord which forms a mathematically harmonic series is better than one that does not do so; the chord C-f-a-c (20, 15, 12, 10) is therefore superior to C-e-g-c (30, 24, 20, 15) [31]. The third supposes that large intervals are more beautiful than small ones; C-c-g' is therefore better than C-g-c [32]. By combining these three principles Papius erects a complicated hierarchy of consonant chords, in which the $^6_4$ occupies an honourable place. He recommends it even as the final chord of a piece [33]; and so uses it himself in the trio, stuffed with $^6_4$ chords, which he gives as an example at the end of his book.

[28] Papius, *De Consonantiis*, p. 69.
[29] Ibid., p. 188: "qua in re vim sensit veritatis, & profitendo laudem aeternam est promeritus".
[30] Ibid., pp. 145-6.
[31] Ibid., pp. 115-143, 146.
[32] Ibid., pp. 147-9.
[33] Ibid., pp. 161, 205.

More interesting, but not more convincing, are the experiments that Papius describes [34]. He had discovered that even well-trained musicians, when trying to sing in unison with a flute, often in fact sang an octave lower. He ordered them to sing a fifth above the lowest note of the flute, and asked them if this made a good consonance. "They praised it greatly. But, without knowing it, they had sung a fourth below". He then made the same experiment with the fourth, which these musicians rejected with contempt, though in reality they had heard a fifth. He then led them to a place where there was an echo which reflected the sounds at their true pitch; he repeated the experiment and the musicians reversed their judgments.

It is difficult to see what these experiments are meant to prove, since, in the first of them, the musicians did in fact, subjectively, hear the flute an octave lower. But it remains true that it is easy to confuse octaves with unisons, and fifths with fourths. This argument is also put forward by the Dutch mathematician Simon Stevin, another very original theorist and defender of the fourth [35].

Mersenne reproduces most of the arguments of the sixteenth-century theorists for and against the $^6_4$; and he confirms, by his own observations, that it is easy to mistake a fifth for a fourth and *vice versa* [36]. But his general intention is to justify the practical rule, although he gives a "fantaisie en faveur de la quarte" [37], which in fact does not contain many $^6_4$s. Doni, who also has studied Zarlino, Salinas and Papius on this question, is a little less conservative. He accepts Zarlino's opinion that a major $^6_4$ is more harmonious than a minor $^6_4$, and points out that in the music played on several instruments, such as the lira da braccio and the guitar, one hears the $^6_4$ "con gran sodisfattione dell'udito". He is

[34] Ibid., p. 54. Papius also suggests a group of experiments (ibid., pp. 60-5), which consist in singing a tune accompanied by parallel consonances, first perfect, then imperfect, and also by parallel $^6_3$ and $^6_4$ chords; that is to say, he is unwittingly reviving the *organum* and *fauxbourdon*. He claims that this is the true test of a consonance, and that perfect parallel consonances are always superior to imperfect ones.
[35] Simon Stevin, *The Principal Works*, ed. Ernst Crone, etc., Vol. 5, Amsterdam, 1966, pp. 450-3.
[36] Mersenne, *Harm. Univ.*, I des Consonances, Pr. xxv-xxxii, pp. 70-80.
[37] Ibid., V de la Composition, Pr. v, pp. 300-3.

therefore surprised that modern composers will not admit this chord as consonant, and he does not believe that it should always be prepared and approached by step. But he confesses that there is "qualche durezza" if the $^{6}_{4}$ is approached by leap; in this case it could be used to express harsh and hard words. He introduces into the argument the phenomenon of beats; which is very rare at this period. If one plays a dissonance on two organ pipes, "one hears a certain beating of sounds that is unpleasant to the ear" ("si sente certo ribattimento de suoni spiacevole all'udito"); one can make the same observation with a lute, if one puts one's ear near the table. The fourth does not produce this phenomenon, therefore it is not dissonant. He ends by recalling the canticles of the Greeks heard by Zarlino and Salinas [38].

* * *

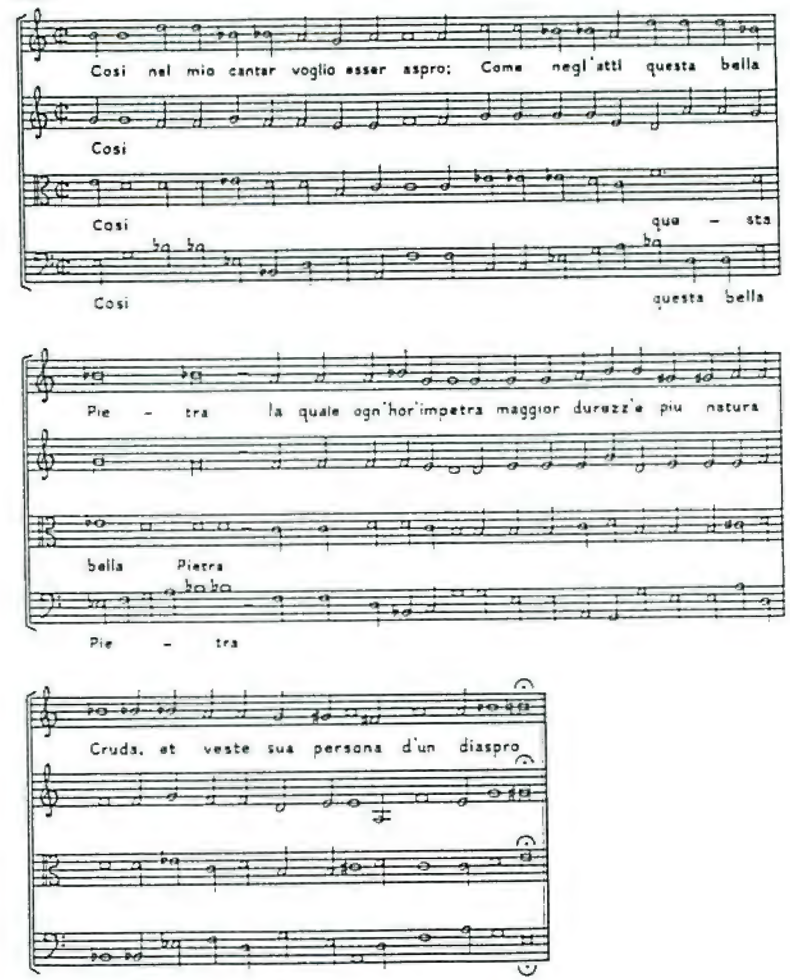
What is the connexion of all these theories with the music of this period? To what extent are they *a priori* constructions, or, on the contrary, derived by induction from the practice of great composers? In one very limited case, that of Mersenne's competition, it is possible, as we shall see in the next chapter, to attempt an answer to these questions; but in the vast field of Renaissance vocal music, to give adequate answers would require years of research and the space of several books. All I can do here is to give a handful of examples as illustrations of some of the main themes of this chapter.

To show the difference between the modern conception of the affective value of the major and minor modes and that of the Renaissance, here is a song which Galilei composed specially to prove that "hardness and harshness" ("il duro & l'aspro") can be expressed without using dissonances and to demonstrate the

[38] Doni, *Annotazioni*, pp. 257-62.

expressive force of false relations [39]:

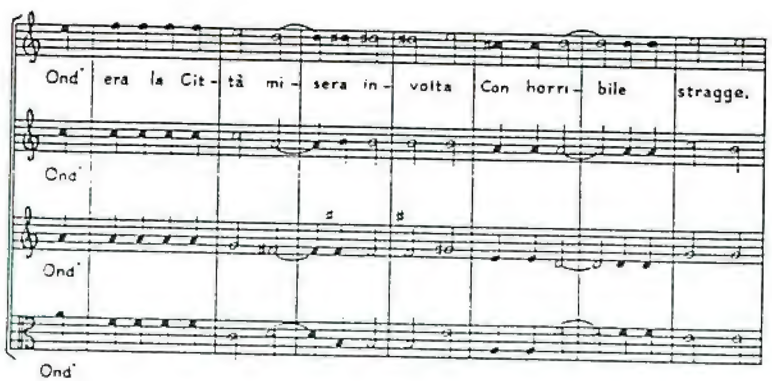
Ex.1

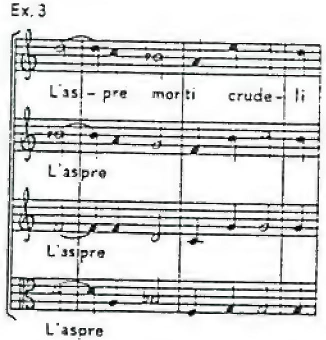


This piece contains only $^{5}_{3}$ chords, mostly major, which, to my ear, give no impression of hardness or harshness, but produce the effect of a calm, solemn, rather mysterious hymn. There is a similar use of major $^{5}_{3}$ chords and of chromatic progressions to

[39] Galilei, MSS. cit., I, 195v-196.

express misery and horror in this passage from one of the choruses by Andrea Gabrieli for the *Edippo Tiranno*, performed at Vicenza in 1585, a humanist enterprise in which it was certainly intended that the music should be expressive of the text [40]:

Ex. 2

Even the defenders of the $^5_4$, such as Doni and Zarlino, believed that it was suitable for expressing hardness and cruelty, and it is so used in these same choruses of Gabrieli, as, e.g. in this remarkable passage [11]:

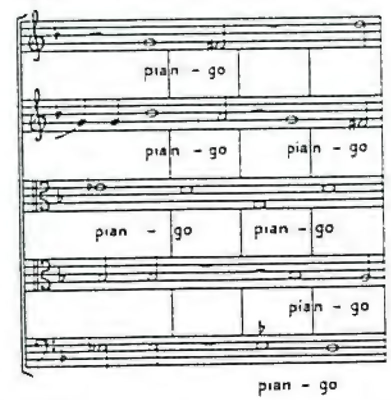
Ex. 3

[40] Andrea Gabrieli, *Chori in Musica . . . Edippo Tiranno . .*, Venice, 1588, in Leo Schrade, *La Représentation d'Edippo Tiranno*, Paris, 1960 (C.N.R.S.), p. 166; cf. ibid., p. 222 "la cruda morte e ucciso".

[11] Ibid., p. 165; cf. pp. 171 "dolenti", 182 "nocive", 191 "infelice", 197 "horror", 219 "offende"; the majority are minor $^6_3$s.

But it must be admitted that quite as often he uses major $^6_3$s, with the third doubled, for misery and horror, as in the passage I have just quoted.

Cipriano de Rore is the madrigalist whom the humanistic theorists cite as the supreme example of the art of expressing the text. Vincenzo Galilei and Giovanni de'Bardi give lists of madrigals that they consider particularly fine in this respect [12]. If one looks at these madrigals, one is indeed struck by his constant care to express the text musically. But the cases where the means employed correspond to the theories of intervals are not very numerous. Nevertheless there are a few, such as these minor $^6_4$s to express lamentation, which are in conformity with Zarlino's recommendations [13]:

Ex. 4



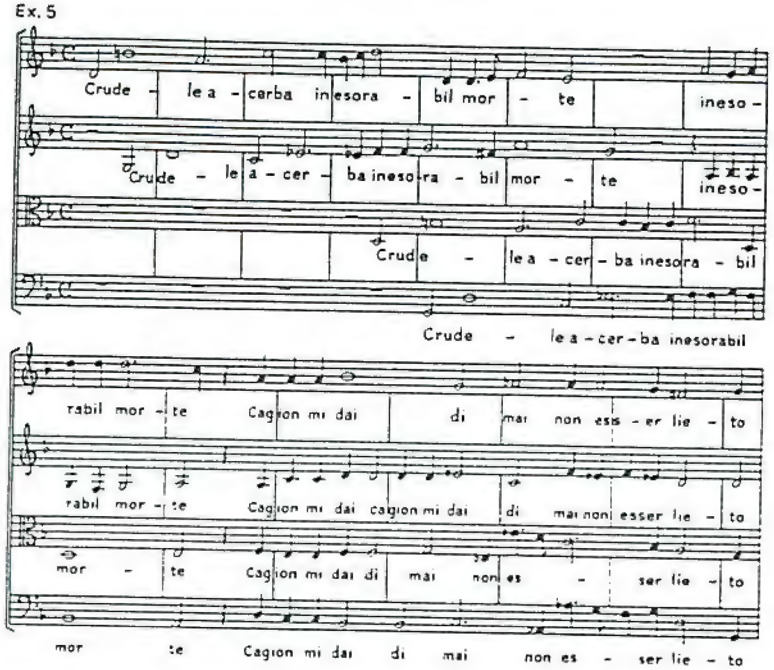
In the following passage the melodic and harmonic major sixths conform to the doctrine of intervals [14]; but one would find nothing in the theorists that would explain the beautiful false relation at

[12] For these madrigals I give in brackets the references for Cipriano Rore, *Collected Works*, ed. Bernhard Meier, Am. Inst. for Mus., 1961- . Bardi, "Discorso Mandato . . a Giulio Caccini", in Doni, *De'Trattati di Musica*, ed. A. F. Gori, Firenze, 1763: Poiche m'invita amore (V, 78); Se bene il duolo (IV, 107); Di Virtù, di costumi (IV, 100); Un altra volta la Germania (IV, 54); O sonno, & della quiete (IV, 66). V. Galilei, MSS. cit., I, 190v: Hor che'l cielo et la terra (II, 4); Anchor che col partire (IV, 31); Cantai mentre ch'io arsi (II, 1); Come havran fin (IV, 34).

[13] Rore, *Works*, II, 6; cf. IV, 66 "Asprezza"; II, 8 "moro".

[14] Ibid., IV, 80; cf. Introduction to Vol II, p. 5 on the use of major and minor chords to "imitar le parole".

"morte", or the final Neapolitan cadence:

Ex. 5

To express harshness and pain Cipriano very often uses a *faux-bourdon* of $^6_3$s, a procedure of which Theodor Kroyer has given us many other examples [15]; and this means of expressing the text is also lacking in the theorists.

If this enquiry were taken further in Cipriano's works, and those of other great madrigalists, I think that it would be found that all these theories of intervals are too simple and crude to be of much help in interpreting the music of this period. But nevertheless the groups of emotions attributed to the major and minor modes, compared with those prevalent since the eighteenth century, are, I think, of considerable importance and would repay further research.

[15] Kroyer, "Die threnodische Bedeutung der Quart in der Mensuralmusik", in *Bericht über den Musikwissenschaftlichen Kongress in Basel*, Leipzig, 1925, pp. 231-42.

CHAPTER VI

MERSENNE'S MUSICAL COMPETITION OF 1640 AND JOAN ALBERT BAN

On 5 May 1640 Joan Albert Ban (1597-1644)[1], a Catholic priest of Haarlem, received, forwarded by Constantijn Huygens, a copy of a little French poem, which Mersenne had sent him [2]:

> Me veux tu voir mourir, insensible Climaine?
> Viens donnér á tes yeux ce funéste plaisir!
> L'excéz de mon amóur, et celúy de ta haine,
> S'en vont en un momént contentér ton desir.
> Mais au moins souviéns toy, cruélle,
> Si je meurs malheureúx, que j'ay vescú fidélle [3].

In the original, by Germain Habert, the first line ended: "trop aymable inhumaine". The arrangement had been made that Ban should set these lines to music and send his song to Mersenne, who would meanwhile have them set by a French composer; various experts in France were then to judge which setting was the better. Ban composed his music very rapidly ("horulae spatio") and dispatched it to Mersenne the following day [4].

The competition was not a fair one. Mersenne had chosen a poem that had already been set by the greatest French composer of the time, Antoine Boësset, and he had sent Ban a slightly, but significantly altered version of it. It was not even a genuine competition at all, since Mersenne had never had any doubt about who was to win it. Writing to another correspondent in Holland, André Rivet, Mersenne asked him to forward a letter to Ban as quickly as possible [5],

[1] On Ban see the article by Frits R. Noske in *MGG*, Bd. 15 (Supplement, 1973), cols. 445-6, and the literature cited therein.
[2] Mersenne, *Corresp.*, IX, 321-3.
[3] Ibid., X, 21; the accentuation is Ban's, cf. infra p. 98.
[4] Ibid., X, 36.
[5] Ibid., IX, 450: "affin que nous achevions le petit combat harmonique que

> so that we may finish the little harmonic combat we have begun which consists in making him see that our songs and compositions are better and more elegant than his.

The reason why Mersenne wanted to teach the Dutchman this lesson can be learnt from their earlier correspondence. They had been put into contact with each other by Constantijn Huygens some time before 1638 [6], and their acquaintance began, on Mersenne's side, with contempt, and, on Ban's, with truculent dogmatism. By January 1638 Mersenne had read Ban's first published work on music, the *Dissertatio epistolica de Musicae Natura . . .* (1637) [7], and found in it nothing that was not "fort trivial"; Ban had apparently read nothing but Zarlino, and, if he had the sense to read Mersenne's *Harmonie Universelle* or *Harmonicorum Libri*, "he would learn more new theory than he has ever conceived" [8]. By May of the same year Ban had sent some of his compositions to Mersenne, and had presumably received an adverse judgment on them and on a statement of his theory of emotive music, *musica flexanima*, which, as we shall see, rested mainly on the belief that melodic intervals have specific emotional effects. Before defending his music in detail, Ban pronounced with angry defiance [9]:

> Whoever does not understand from [mathematical] proportion the particular power of intervals, as described by me, is raving mad; whoever does not distinctly perceive these powers by ear, when they are applied to words, is a deaf musician and an "ass at the lyre".

nous avons commencé, qui consiste à luy faire voir que nos chants et compositions vallent mieux et sont plus elegantes que les siennes".

[6] Ibid., VII, 1-2.

[7] Joannes Albertus Bannius, *Dissertatio Epistolica, De Musicae Natura, Origine, progressu, & denique studio bene instituendo*, Leiden, 1637. This work also appeared in: *H. Grotii et aliorum Dissertationes de studiis instituendis*, Amsterdam, 1645, and *Gerardi Io. Vossii et aliorum Dissertationes de studiis bene instituendis*, Utrecht, 1658.

[8] Mersenne, ibid., VII, 34 (letter to Rivet): "S'il a l'esprit de voir le livre, soit françois ou latin, lequel j'ay fait de ce sujet, il apprendra plus de nouvelle theorie qu'il n'en a jamais conceu".

[9] Ibid., VII, 206 (letter from Ban to Mersenne): "Quisquis intervallorum propriam potestatem a me descriptam non intelligit ex proportione, ille delirat; qui non distincte eas auribus percipit dum pronunciantur ad verba, ille surdus est musicus, καὶ ὄνος πρὸς λύραν". Cf. Ban reporting and agreeing with Descartes' disparaging remarks about Mersenne in a letter to Huygens of October 1639 (in *Correspondance et oeuvre musicales de Constantin Huygens*, ed. Jonckbloet & Land, Leiden, 1882, p. lxvi).

In spite of this acrimonious start the two musicians continued to correspond, and it is not surprising that Mersenne, faced with the stone-wall of Ban's unshakeable confidence in his own music and theory, should wish to break down the wall by means of a practical and public test. But, before discussing the competition, I must outline Ban's musical theory, so that the reader may understand the principles on which he composed and on which he judged other people's compositions.

Our knowledge of Ban's musical theory and practice is incomplete and fragmentary, since his projected full-scale treatises in Latin and Dutch either were never completed or have not survived [10], and of his one musical publication, the *Zangh-Bloemzel* of 1642, only the alto and basso continuo parts are now available [11]. Thus of his compositions we have only this incomplete work and the competition song, and of his theoretical works only the *Dissertatio*, the preface to the *Zangh-Bloemzel*, a *Kort Sangh-Bericht* of 1643 [12] and letters, which are easily available in the correspondence of Mersenne and of Constantijn Huygens. It may be, then, that his theories would be more convincing if we possessed a fuller exposition of them, and his compositions based on these theories more impressive if we had more than this fragmentary acquaintance with them. But to judge from what does survive, this is unlikely.

Ban had from his youth studied music and composed, but without formal instruction [13]. By 1636 he had written the *Dissertatio* and was acquainted with Huygens and Descartes [14]. Through these friends he soon became a correspondent of Mersenne, and, through Mersenne, of Giambattista Doni [15]. But these distinguished musicians had no influence on the impervious Ban, whose theories remained unchanged by them, except that he took over

[10] See article cited above, note 1.

[11] Ban, *Zangh-Bloemzel (Theoretical Part)* & *Kort Sangh-Bericht*, fascimile ed. Frits Noske, Amsterdam, 1969.

[12] See previous note.

[13] Ban, *Zangh-Bloemzel*, "Tot den Leezer" (no pagination); *Dissertatio*, p. 11.

[14] See Huygens, *Corresp.*, pp. xxxviii-xxxix; Mersenne, *Corresp.*, VII, 1-2.

[15] See Mersenne, *Corresp.*, VIII, 341.

Descartes' division of the octave into eighteen unequal steps. Ban claimed to be entirely original in his musical theory, and the claim is justified in the sense that he did not consciously follow any earlier tradition or contemporary models and that he evolved his system largely by solitary, *a priori* reasoning. It is difficult to be original without being silly and wrong; we must then not be surprised if Ban turns out often to be both—in our eyes, and in the eyes of his contemporaries.

But of course it is impossible to be entirely original, and Ban, like Descartes, was not wholly successful in demolishing the old house before he built his new one [16]; there are important traditional elements at the base of his system. From musical humanism he took over the following cardinal assumptions [17]: that the chief aim of music is to produce emotional and ethical effects; that these effects are produced mainly by the text, to which the setting must be entirely subordinated; that for such *musica flexanima* the system of intonation used is of the greatest importance. Ban's main source for these assumptions was, as Mersenne noticed, Zarlino [18], though he also cites some earlier musical humanists, notably Gafori and Glareanus [19], and had some direct knowledge of ancient musical theorists, at least of Boethius [20]. But he differed sharply from earlier musical humanists, such as Zarlino or Vincenzo Galilei, and from contemporary ones, such as G. B. Doni or Mersenne in his *Quaestiones in Genesim* period, in denying that ancient music did in fact achieve these effects or was in any respect superior to modern music. He dismissed the classical accounts of the effects of music, even such time-honoured stories as that of Timotheus and Alexander, as old wives' tales ("aniles fabulas & stultorum commenta") [21] or as due to the

[16] Descartes, *Discours de la Méthode*, 2e Partie (*Oeuvres et Lettres*, ed. de la Pléiade, Paris, 1949, p. 100).

[17] See Walker, "Musical Humanism".

[18] Ban, *Diss.*, pp. 29-30, 57-9.

[19] Ibid., pp. 39, 33; in 1640 he read, and approved of, Salinas (*Kort Sangh-Bericht*, pp. 17-8).

[20] *Diss.*, p. 25.

[21] Ibid., pp. 36-7.

boasting of *Graecia mendax* [22]. He regarded the history of music as one of continuous improvement up to his own day, and particularly up to his own work. Unlike Doni, he believed that the Greeks had no polyphony, since they did not use the imperfect consonances, and that therefore their music was very simple and rudimentary [23], though he was far from realizing, as did Kepler, what an extraordinary leap forward the discovery of polyphony was. The real progress, according to Ban, began in the Middle Ages, some time after Guido d'Arezzo, when the imperfect consonances were discovered, "whence this art, fuller of majesty and delight, has been brought to a nobler perfection" [24]; and he confidently asserts that modern music will surpass the music of the ancients by an infinite distance [25]. With the dismissal of ancient music went a rejection of the modes and genera [26], two of the major preoccupations of most musical humanists, though Ban was interested in chromaticism in the modern sense, and attached great importance to the various diatonic species of the octave.

Without the imitation of antiquity as a guide, how then was modern music to achieve the effects? Ban found his answer in the second of the assumptions mentioned above: the predominance of text over music. Music must become a kind of oratory [27]:

[22] Ban, *Zangh-Bl.*, "Tot den Leezer". In a letter to Huygens of January 1641 (Huygens, *Corresp.*, p. cxv) Ban does, very exceptionally, invoke the authority of "Plato caeterique sapientes" for the ethical power of music; but he immediately adds: "Sed ab authoritate ductis argumentis mihi aliquid asserere non sufficit: nisi et rationibus mens convincatur".

[23] Mersenne, *Corresp.*, VIII, 342 (letter from Ban to Doni of March 1639); Ban, *Diss.*, pp. 20-2.

[24] Ban, *Diss.*, p. 28: "unde ars ipsa plenior majestatis ac delectionis ad perfectionem nobiliorem deducta est".

[25] Ibid., p. 36: "ingenue dico, & religiose assevero, Musicam hanc nostram, Graecorum illam, & priscorum Latinorum alteram, infinitis parasangis sua perfectione, & efficacia excessuram".

[26] See, e.g., Ban's letter to Doni cited above, note 23.

[27] Ban, *Diss.*, pp. 33-4: ". . . eo quod sonitus verborum ac syllabarum quantitati, significationi, & sensui aptandus sit, ut energiam habeat Musica, Rhetorisque munus subeat, docendo, delectando, & movendo". Cf. Huygens, *Corresp.*, pp. lxiii-lxiv (letter from Ban to William Boswell of January 1637), cxv (letter from Ban to Huygens, January 1641: "omnem Musicae perfectionem magis in motione animi, quam delectatione constitutam esse. Deinde movere animos plus congruere Musici quam Rhetoris officio seu facultati. A musica enim mecum omnes antiqui natam oratoriam artem statuunt").

> the musical sound must be fitted to the quantity, meaning and sense of the words and syllables, so that Music may have energy, may take over the function of the Orator, in teaching, delighting and moving.

It is worth remembering in this context that Cicero, in the *De Oratore*, which Ban cites, applies Ban's favourite term *flexanima* to oratory [28]. Like rhetoric, then, music must have rules that can be taught, and these rules must be deduced by reason from first principles [29], not induced from the practice of great composers, since, with the possible exception of Monteverdi [30], no composer has yet achieved the true *musica flexanima*. Just as Ban wishes, like Descartes, to accept nothing from the past, so he also wishes to owe nothing to any happy accident, to any instinctive drive, to any unconsciously acquired skill; his system of composing must be deduced from clear and distinct ideas, and his music must produce its effects not by chance (*casu*), but by necessary, scientific rules [31].

With regard to the relation of text to music, Ban, then, starts from the extreme humanist position: "All the power of this [soul-moving] song is in the utterance of the words" [32]. The first, most basic rule is therefore that the text must be intelligible, and this can only be ensured, in polyphonic music, by syllabic homophony, such as the composers of *musique mesurée à l'antique* practised [33] (though Ban never mentions these) [34]:

[28] Cicero, *De Oratore*, 2, 44, 187; Mersenne, *Corresp.*, X, 23-4 (letter from Ban to A. M. van Schuurman).

[29] See, e.g., Mersenne, *Corresp.*, X, 403-4 (Ban to Mersenne, January 1641).

[30] Ban, *Zangh-Bl.*, "Tot den Leezer".

[31] Mersenne, *Corresp.*, X, 114 (Ban to Huygens, September 1640): "Unde nihil a me casu vel fortuito, sed ex certissima scientia fit quicquid in musurgia geritur"; cf. ibid., X, 29, 35, 40 (Ban to A. M. van Schuurman, August 1640).

[32] Ban, *Kort S.-B.*, p. 3: "Alle de Kracht van deezen zangh is in het uitspreken der woorden".

[33] See Walker, "Musical Humanism", ii, 308.

[34] Mersenne, *Corresp.*, X, 32 (Ban to A. M. van Schuurman, August 1640): "cùm oratio musica vel cantu, vel concentu recitanda, sensibiliter ab auditoribus intelligi debeat: a ratione pulchri longè alienum est, si a diversis vocibus diversa verba vel diversae syllabae eodem temporis momento recitentur. Istud enim enormiter verborum perceptioni intellectuique officit, vitiumque est passim omnibus usitatum, quod fere nemo advertit . . . Et haec quidem generalis regula esto, passim, hoc est ubique, et semper, religiose observanda, *ne una vox alteram in verborum pronunciatione turbet*".

> Since musical speech, either monodic or polyphonic, ought to be clearly understood by the listeners, it would be very far from the rule of beauty if different words or different syllables were pronounced by different voices at the same moment. For this is enormously obstructive to the perception and understanding of the words, and this vicious practice is universal, though noticed by hardly anyone . . . . Let this then be age neral rule, always and in every case to be religiously observed, *that one voice shall not obscure another in the pronunciation of the words.*

Ban also follows traditional musical humanism in demanding that the music shall reproduce the spoken rhythm of the text. But, owing to his contempt for antiquity, he is not interested in poetic metre, and certainly has no thoughts of reviving ancient quantitative metres, as did Baïf and his musicians or some English humanists. Unfortunately Ban's conception of verbal rhythm was exceptionally confused. It was usual to confuse quantity and stress, as did the humanists who revived ancient metres, that is, they were not consciously aware of the stress accent of modern languages, but tended nevertheless to put a stressed syllable where the ancient metre demanded a long [35]. But Ban went further by consistently failing to distinguish between quantity, stress *and* pitch accent. This is evident both from his directions in the *Kort Sangh-Bericht*, where he states that long and short syllables "are to be expressed by the rising and falling of the voice, or at least by the number of the time [i.e. the length of the note]" [36], and from his comments on the competition airs.

Ban lays great emphasis on the importance of the melody, which must be given to the highest voice, since this is the first to pierce into the ears and move the imagination (he seems here to be reviving the curious error of Plato [37], that high sounds travel faster than low ones) [38]:

[35] See Walker, "Musical Humanism", ii, 303-4, and "The Rhythm and Notation of *Musique Mesurée*", *Musica Disciplina*, IV (1950), pp. 181-2; Augé-Chiquet, *J. A. de Baïf*, Paris, 1909, Ch. VIII.

[36] Ban, *Kort S.B.*, p. 4: "welke werden uitgedrukt met het klimmen en daelen des stems: ofte ten minsten met het getal des tyds".

[37] Ibid., p. 3; Plato, *Timaeus*, 79 E-80C.

[38] Ban, *Diss.*, p. 33: "Modulamentum, Idea, ac πρωτότυπος Musicae est, cui

Melody is the Idea and Prototype of Music, to which the other voices are to be fitted. In the invention and constitution of melody lies the whole business of music.

Nevertheless, he is not advocating monophony, or even instrumentally accompanied solo—when he tried out his composition air he had it performed polyphonically [39]. But, though he was aware of the expressive powers of harmony and had some interesting things to say on the subject, his main attention was directed towards melody, and it is for the expressive qualities of melodic intervals that his system provides the most detailed theory and instruction.

According to Ban, melodic intervals have a specific, individual power of moving the soul ("specificam individuamque quandam habent movendi potestatem") [40]. He deduces these powers from the general principle that smaller intervals strike the ear more gently ("mollius"; in the *Kort Sangh-Bericht*, "lieflyker") and large intervals more violently ("vehementius"; "heerlyker, ende geweldiger") [41]. These intervals should, therefore, affect the feeling and imagination in a continuous gradation of emotions, from gentle-sweet-sad ("blandus") to violent-harsh-angry ("vehemens"), corresponding to the increasing size of the intervals. But Ban then takes the semitone, *blandus*, and the tone, *vehemens*, as elements and grades the other intervals according to the number of tones or semitones they contain in the diatonic scale. This complicates the simple correspondence of size of melodic interval to degree of blandness or vehemence; for example, a minor third is blander than a tone because it contains a semitone, although it is a larger interval, and a minor sixth is blander than a fifth because it contains two semitones; but it is not clear whether a minor sixth,

concentus vocum aptandus. In hujus inventione ac constitutione tota res Musica versatur".

[39] See Mersenne, *Corresp.*, X, 36.

[40] Ban, *Kort S.B.*, p. 7; Ban apud Mersenne, *Corresp.*, X, 22-3.

[41] Ban, *Kort S.B.*, p. 7, writes as if intervals were one sound, not two, stating that every sound is a pulse of the air ("een slagh des luchts") caused by a sounding body ("klinkendt lichaem"), which affects the ear according to the size of the body—semitones have the smallest body, therefore, etc.

with two semitones and three tones, is blander than a minor third, with one semitone and one tone. However, from Ban's remarks on the choice of species of the octave one can construct a scheme in which the bland intervals, in order of decreasing blandness, are the semitone, the minor third and the minor sixth, while the vehement intervals, in order of increasing vehemence, are the tone, the major third, the fourth, the fifth and the major sixth. The octave, being so nearly a unison, has no specific expressive power; sevenths are not mentioned.

Ban's division of all emotions into two broad classes, and the ascription to these classes of two groups of intervals which, for the thirds and sixths, correspond to our modern major and minor modes, probably had their origin in Zarlino. But Zarlino, as we have seen, was less crudely dogmatic, and he also had the merit of considering harmony at the same time as melodic intervals [42]. In the *Kort Sangh-Bericht* Ban does briefly relate his scheme of melodic intervals to harmony, simply by stating that their characteristics are the same when sung or played as chords [43]. Since this cannot possibly apply to the tone and semitone, and since by this time a chord of a bare fifth and octave is rare, we are left with the thirds and sixths, and the same rough correspondence, as for melodic intervals, between vehement and major mode, and bland and minor mode, a correspondence which, again, is close to the one suggested by Zarlino. This is confirmed by Ban's remarks on consonances in his long letter on the competition to Anna-Maria van Schuurman [44]. The following emotional qualities are assigned to the consonances, mostly without any explanation for which we are referred to Ban's Latin treatises, now lost:

minor third: soft, bland and languid ("mollis, blandus et languens")
major third: energetic ("incitatus")
fourth: harsh ("tetrica"), because it cannot be divided into two harmonic intervals
fifth: heroic and martial

[42] V. supra p. 69.

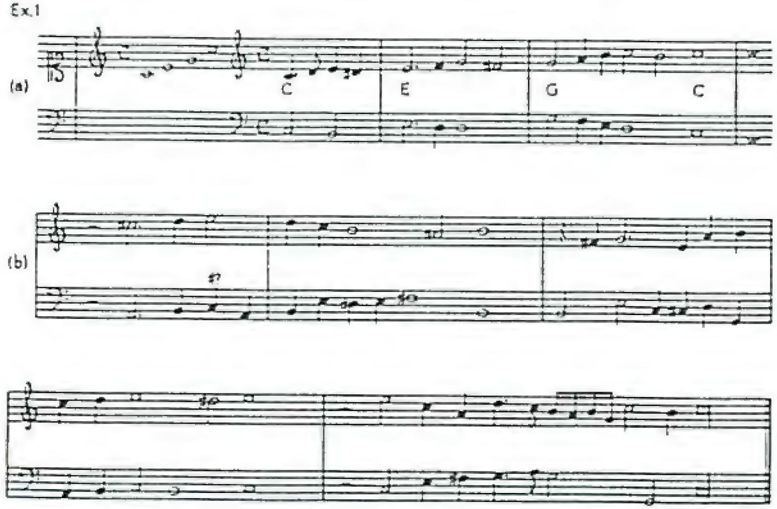
[43] p. 7.

[44] Mersenne, *Corresp.*, X, 30, 33.

minor sixth: more flattering and languishing ("blandiens languensque") than the minor third, because it is a wider interval
major sixth: severe and expressing weight or bulk ("molem")
octave: merely pleasing, has no power of moving.

Dissonances also have specific powers of affecting the emotions, but, unfortunately, for these we are again referred to the Latin treatises.

More interesting, though also more obscure, are Ban's recommendations of the use of modulation for expressive purposes [15]. After outlining the above system of the expressive qualities of melodic and harmonic intervals, he states that these qualities are much more powerful if the music goes outside its "circle", i.e. the species of the octave in which it begins and ends—in modern terms, if it modulates away from the tonic key. (Ban, I think, uses the term "circle" (*circulus*, *Zanghkreits*) to avoid involving himself in mediaeval or ancient modes). He then gives examples of such modulations, of which I reproduce the first, in C [16]:

Ex.1

From these one can see that he regards the primary modulations as being to the keys of the third and fifth degrees of the scale

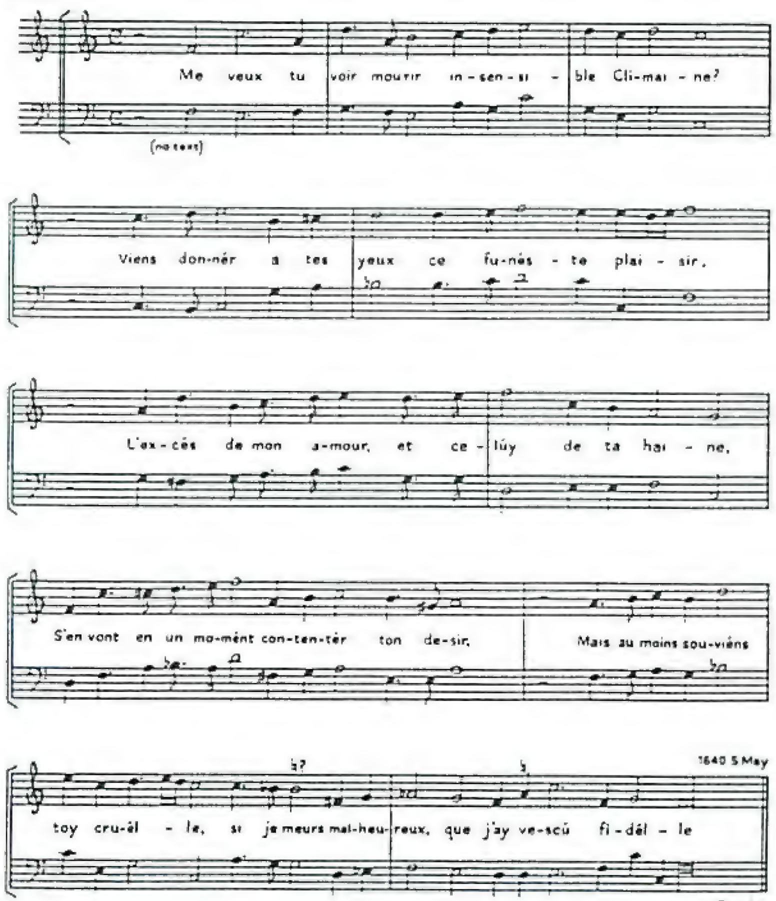
[15] *Kort S.B.*, pp. 7-11.
[16] Ibid., pp. 8-9. He gives other, monodic, examples for D, E, F, G, and A.

(Ex. 1a), but also recommends further modulations, in his first example to the very remote key of the leading note (Ex. 1b). Ban asserts enthusiastically that these modulations produce wonderful effects on the emotions ("wondere ontroeringhe des gemoedts"). But the only hint he gives of how or why they do so is the remark that, as sweet food tastes sweeter after bitter, and *vice versâ*, so these contrasts of sadness and joy are effectively moving. It is not clear whether he is here referring to the contrasts of major and minor that occur in many of these modulations, or perhaps to the feelings of tension and relief produced by a modulation to a remote key followed by a return to the tonic—in the first example he describes the return from B (major or minor?) to C major, via an interrupted cadence in E minor, as the music being "brought back again to its old way" ("wederom gebracht tot zyn oude gangh").

We need not examine in detail Ban's theories about intonation, since these are only marginally relevant to the competition. Like Doni, he believed that a correct system of intonation was of the greatest importance for the production of the effects [17], and, like the great majority of later sixteenth- and seventeenth-century theorists, he was in favour of just intonation. In the *Kort Sangh-Bericht* he gives a diagram of a keyboard [18] which, by means of six extra keys, represents Descartes' eighteen-step division of the octave, and which would enable one to play in just intonation in the keys then most in use (from C major to D major and B flat major), though his own suggested modulations go outside this range. Ban evidently put this theory into practice, since in 1639 he had a harpsichord constructed in this manner, and also lutes and viols so fretted, and he gives sensible directions for playing on such an instrument, tuning it, and using it to train singers to distinguish between such intervals as the major and minor tone, diatonic and chromatic semitones and so on [19].

[17] Ibid., pp. 44-5.
[18] Ibid., p. 28.
[19] Ibid., pp. 19-27, 38, 43-4; cf. Mersenne, *Corresp.*, VIII, 536-8.

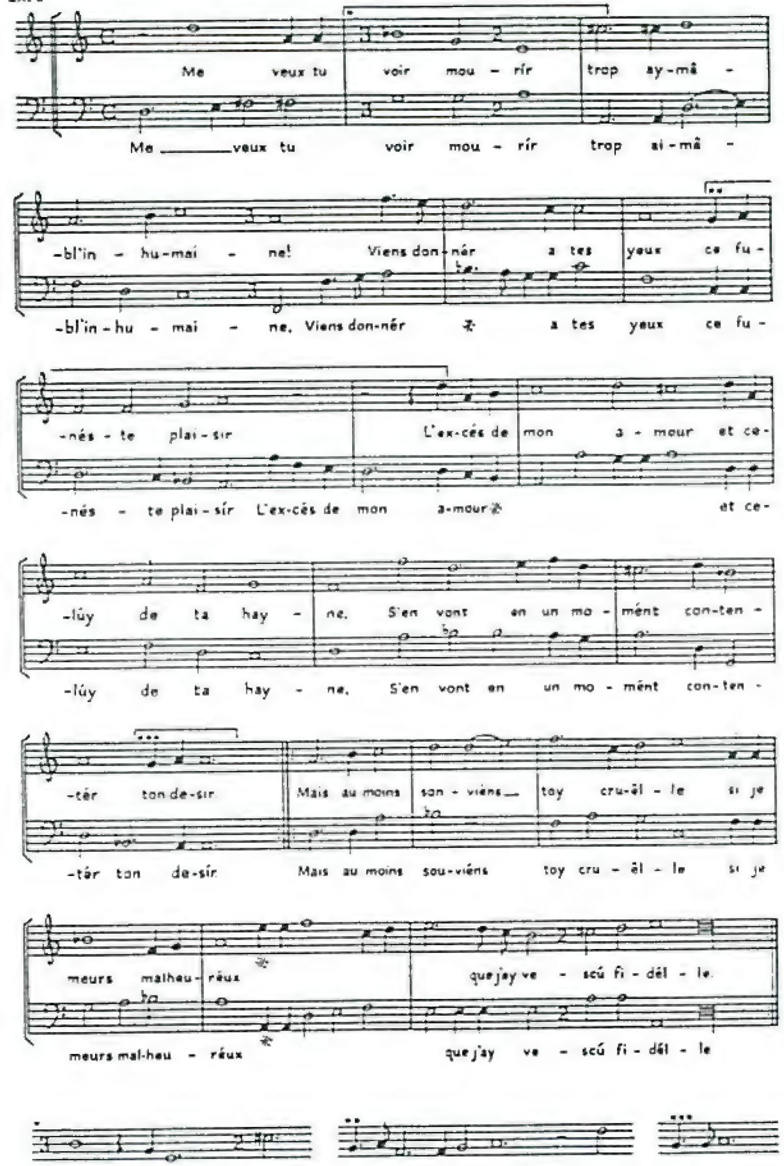
Let us now return to the competition [50]. I give here the two airs in two-part versions as quoted by Ban in his letter to A. M. van Schuurman [51]. It is unfortunate that for Ban's air (Ex. 2) we have no fuller version, since it is by no means always evident from just the treble and bass what harmony he intended, whereas for Boësset's air (Ex. 3), apart from his harmonic progressions being much clearer, we have the published polyphonic version [52].

Ex. 2

[50] On this, in addition to works already cited, see André Pirro, *Descartes et la musique*, Paris, 1907, pp. 109-20.

[51] Mersenne, *Corresp.*, X, 36, 31 (variants from p. 24).

[52] *IXe Livre d'airs de cour à quatre et cinq parties*, Paris, 1642 (see *RISM*, A/i/1.353, and Mersenne, *Corresp.*, IX, 323 n. 3).

Ex. 3



In the opinion of everyone who saw or heard the two airs Boësset's was undoubtedly by far the better setting, the only exceptions being Ban himself and some unnamed Dutch experts, whom he mentions vaguely and evasively [53]. Given the unfair conditions of the competition this result is not surprising. And we must remember that Ban and his critics were judging the airs on radically different criteria, as Ban himself complains [54]—indeed this difference constitutes the main historical interest of the competition. For Ban the excellence of a musical setting lay solely in its reinforcing the emotional content of the text in such a way that the audience should feel the same emotions. His opponents, on the other hand, whether explicitly, as in the case of Mersenne, or, more usually, implicitly, believed that the prime function of a song was to give pleasure and that the fitness of the music to the emotional content of the text, though perhaps important, was only part of the total pleasurable experience or of the general excellence of the song.

But even if Ban's opponents had accepted his criteria of judgment, he would still have lost the competition, because, according to them, he had misunderstood what emotions the poem was meant to express. Ban took its overall emotional tone to be one of indignation and anger, tempered at times by sadness [55]. His opponents (and, it appears, the author of the poem) claimed that it represented amorous pleading, a lover trying to wheedle his cold mistress into a more yielding state of mind [56]. In terms of Ban's two classes of emotions, he read the poem as vehement, they as bland. One can now see why the alteration of the first line seemed important to Ban [57]; "trop aymable inhumaine" is unmistakably amorous and wheedling, whereas, with "insensible Climaine", it is certainly possible to read the verses as expressing indignation at long fidelity unjustly rewarded by constant cold-

[53] Mersenne, *Corresp.*, X, 401 (Ban to Mersenne, January 1641).
[54] Huygens, *Corresp.*, p. cvi (Ban to Huygens, January 1641).
[55] Mersenne, *Corresp.*, X, 23.
[56] Ibid., X, 48 (Villiers to Mersenne, August 1640), 238 (Mersenne to Huygens, November 1640); see also X, 242.
[57] Ibid., X, 21.

ness. Indeed it was not only Ban who thought the alteration important. A friend of Mersenne's, Christophe Villiers, who was an amateur composer, had set the original version to music; when he saw the altered version, he found his setting "trop mol" and wrote another, more indignant one [58]. Ban therefore had good reason for complaining about the alteration, and for finding Mersenne's first summary judgment on his air [59]—that it was not as "patheticus" as Boësset's—off the point, since his air was meant to be, not pathetic, but indignant. But perhaps the alteration was not really so important to him, since he persisted in his own interpretation of the poem even when examining Boësset's setting of the original version, which he constantly criticized for not being indignant enough.

The difference between indignant-vehement or wheedling-bland will evidently affect the entire character of the setting, and this character is, according to Ban, first and foremost determined by the "circle" or mode used. He therefore criticizes Boësset for choosing the "circle" D-f-a-d, which, because of the minor third D-f, is "light, soft and bland", and congratulates himself for using the "circle" F-a-c-f, which has the vehement major third F-a and the most "powerful" kinds of minor third, a-c, fourth, c-f, and fifth, F-c, and is therefore eminently suitable to indignation [60]. These kinds of the latter intervals are powerful, it seems, because they have their tones at the bottom and their semitones at the top—a new, and unexplained, principle of grading intervals.

Boësset, replying to Ban's criticism in a letter to Huygens, agrees that, if the character of the poem were such as Ban supposed it (which he denies), then "I should have been wrong in choosing the mode I used, which is full of sweetness and more fitted to express sorrow and complaint than anger" because of the minor third of the tonic chord (D minor), and he would have used the

[58] Ibid., X, 48-9.
[59] Ibid., X, 20.
[60] Ibid., X, 25, 37.

same mode as Ban, "which is the seventh mode [in Zarlino's system], suitable to fury, shrieks and despair, because of the major third which is as virile as the minor third is soft" [61]. Ban was overjoyed that Boësset should agree with him on the character of these two modes, and of major and minor thirds; he venerates his "most exquisite judgment in musical matters", and exclaims: "Oh most excellent musician, how proud I am that he should agree with me on this point! For not all musicians penetrate these mysteries so deeply" [62]. Ban's exultation over this point of agreement seems really remarkable when one knows that in the same letter Boësset had treated Ban's air with deliberately offensive contempt [63]. But Ban was right to rejoice, since this is one of the few positive points of any interest to emerge from the competition and its controversies, namely, as we have seen in the last chapter, that there was general agreement that the character of a mode was determined by whether the third above the keynote was major or minor, and that the major modes were vehement and the minor ones bland.

But Boësset went on to say: "If I want to, I can express any kind of passion as well in one mode as in another", and to explain that the use of accidentals could make them all the same, or at least differ only at the beginning and end of a piece [64]. From what follows it is clear that Boësset is thinking of modulation and not

[61] Ibid., X, 251-2: "j'aurois faict un mauvais choix du mode dont je me suis servy, qui est plain de douceur et plus propre à exprimer la douleur et la plainte que la cholere, son diapason estant divisé dans la cadence par la tierce mineure, et sa replique estant patetique et molle. C'est pourquoy j'ay eu raison de me servir dudict Mode pour esmouvoir à pytié la personne aymée; que sy la signification des parolles eussent esté telles que ledict Sr Bannius se l'est imaginé, je me serois servy du Mode dont il a composé le sien, qui est du 7e Mode, propre à la furie, aux crys et au desespoir à cause de la division de la tierce majeure qui est aussy virile que la tierce mineure est molle . . .".

[62] Huygens, *Corresp.*, p. xcvii (Ban to Huygens): "In hoc porro laudo D. Boëssetum illiusque exquisitissimum in re musica judicium veneror, cum mihi consentiens inquit: . . . . O Musicum excellentissimum, qualem glorior ea in re mihi consentientem! Omnes enim musici ista mysteria tam profunde non penetrant".

[63] Mersenne, *Corresp.*, X, 253.

[64] Ibid., X, 252: "bien que, quant il me plaira, j'exprimeray toute sorte de passion aussy bien en un Mode qu'en l'aultre. Et c'est une erreur de croire le contraire, les accidentz, desquels l'on se peut servir avec addresse, les rendant tous esgaulx; et je soustiens qu'il n'y a que le commencement et la fin qui les rend dissemblables".

of making two modes exactly equivalent except in pitch, as in our modern key-system. For Boësset, then, a difference between modes does survive, but tends to reduce itself to the distinction between major and minor. This awareness of the incipient breakdown of the modal system, and of one of its symptoms, the free use of accidentals, is also shown by other commentators on the competition, Mersenne and Jacques de Gouy, who claim that Ban's remarks on the choice of mode are therefore irrelevant. But Mersenne too is thinking of modulation rather than anything like our key-system [65].

Ban, however, commenting on Boësset's letter, is thinking of making modes exactly equivalent [66]. He argues against doing this on the grounds that, in his system of intonation, namely just, this cannot in fact be achieved, observing that in the case of C major and D major, for example, the order of major and minor tones at the beginning of the two modes will still be different [67]. He shows considerable perspicacity in implying clearly that this complete levelling out of the modes, i.e. anything like the modern key-system, would only be possible with the use of equal temperament, which he rejects, like the majority of his contemporaries, on the grounds that all melodic intervals and consonances would be false and thereby lose their "energy" and specific characters [68]. He agrees, as we have already seen, with Boësset on the great value of the use of modulation.

The greater part of Ban's criticism of Boësset's air and justification of his own is taken up with the question of accents, about which, as already noted, his ideas are even more confused and erroneous than those of his contemporaries. His theory, in so far as one can make it out, is that accents in spoken French are

[65] Ibid., X, 205, 238-9, 260.

[66] Huygens, *Corresp.*, p. xcviii (Ban to Huygens).

[67] Ban's order is the less usual: C-D minor tone, D-E major tone. As his eighteen-step keyboard had two D naturals, this particular example is not convincing; but his argument would apply to other keys, e.g. C major compared with F or A major.

[68] Ibid.: "Si vero utamur systemate imperfecto ac vulgari (ubi omnes toni et semitoni sunt aequales) peribit energia et varietas constitutionis, omnesque consonantiae erunt inconcinnae, hoc est, superfluae, vel diminutae etc.".

primarily musical, like those of ancient Greek, from which indeed their names derive. He mentions acute, grave and circumflex accents, but in practice deals only with acute accents, which indicate a rise in pitch. But he appears to believe that accents also indicate the quantity of syllables, an acute accent corresponding to a long. In practice he puts acute accents on syllables that bear the tonic stress (in French, a very weak one), as one can see from his version of the competition poem, given above. The practical consequences of this theory are that Ban wishes every stressed syllable to be accompanied by a rise in the melody, or at least by a long note, or preferably by both. This principle sometimes comes into conflict with his theory of expressive melody. For example, he criticizes Boësset for setting "mourir" with a falling minor third, whereas the last, stressed syllable should rise; but he has to admit that the falling interval is appropriate to the idea of dying, and that at least the last syllable has a long note [69]. Or again, Boësset should not have set "amour" with a falling semitone, and Ban corrects his setting by making the melody rise a tone [70], forgetting that on his own theory the semitone is eminently bland and soft and therefore suitable to love.

Still more difficulties arise when we consider polyphonic settings. Ban realizes that all the voices cannot always move in the same direction, and therefore concedes that inner voices may observe the "acute accent" (i.e. stress) only rhythmically, by having a long note under the stressed syllable. But he wishes that the bass should rise on a stressed syllable, though admitting that this rule cannot invariably be followed, as it must be by the soprano [71]. The unfortunate consequences of this requirement are evident in Ban's air: the lack of contrary motion between treble and bass, and the long strings of parallel tenths, producing some awkward false relations (at "moment contenter" and "souviens"), which have no apparent expressive purpose.

[69] Ibid., pp. c, cv; Mersenne, *Corresp.*, X, 25.
[70] Ban apud Mersenne, *Corresp.*, X, 25-6, 28.
[71] Ibid., X, 29-30.

Ban's critics were unanimous, and of course correct, in denying that spoken French had a fixed musical accent [72]; but they made no attempt to sort out Ban's confusion of stress with quantity and pitch-accent, nor to provide any alternative theory. Mersenne referred to his own treatise on accents in the *Harmonie Universelle* [73], where in fact he is quite as confused as Ban. Both he and Descartes [74] are at least aware that the patterns of rising and falling pitch in spoken French are extremely complicated and variable, and that these variations are often connected with the emotion expressed. Mersenne also points out the disastrous consequences, noted above, of Ban's theory of accents for part-writing [75].

The two examples given above of Ban's blaming Boësset for setting a stressed syllable with a falling interval, "mourir" and "amour", are singled out by Mersenne, with good reason, as the most beautifully apt musical expressions of single words in the whole of Boësset's air. He notes the appropriateness of the falling diminished fifth for "voir mourir", where the "unpleasantness" of the tritone B♭-E♮ is softened by the two minor thirds. The falling semitone on to C♮, closely preceded by a C♯, for "amour", "charms so strongly that only Marsyas and his like could complain of it" [76]. Descartes also picks out the "elegance" of this setting of "amour": "its last syllable is softened by a semitone, pathetically" [77].

In Ban's theoretical statements of his system of melodic intervals and their expressive qualities there is a curious omission: we are not told whether a given interval has the same or a different quality when rising or falling, or, if there is a difference, what it is; and this, as we have seen, was one of the guiding principles of

[72] Ibid., X, 205, 239, 244-7.
[73] Ibid., X, 239-40.
[74] *Correspondence of Descartes and Constantyn Huygens*, ed. Leon Roth, Oxford, 1926, pp. 294-6.
[75] Mersenne, *Corresp.*, X, 240, 244.
[76] Ibid., X, 242, 246 ("La derniere sillabe d'*amour*, qui baisse d'un demiton, et qui charme si fort estant bien chantée, qu'il n'y a ce semble que Marsyas, ou ses semblables, qui s'en puissent plaindre").
[77] Descartes & Huygens, *Corresp.*, ed. cit., p. 295 (Descartes to Ban): "vox inde per gradus ascendat usque ad primam verbi *amour*, in quo etiam elegantia est; et eius ultima semitonio mollitur, pathetice".

earlier theories of intervals. But from his comments on his own and Boësset's air it appears that he assumes that rising intervals are suitable for indignation and energy, and falling ones for weakness, in particular for dying. How these two classes—rising/energetic, falling/weak—fit in with the other two classes—tone, major third, etc./vehement, semitone, minor third, etc./bland—is not at all clear. The rising fifth and fourth at the beginning of Ban's air are, according to him, energetic and indignant, and at the same time vehement, so that there is no conflict between the two sets of classes. But the minor thirds and semitones at "ce funeste plaisir", "contenter ton desir" and "malheureux" are said by Ban to be bland and sad, although they are rising intervals, so that here the second set of classes apparently overrides the first [78]. On the other hand, Ban alters Boësset's rising bass at "voir mourir" to a falling major sixth (g-B♭-A) in order to express the weakness of dying, although the major sixth is a highly vehement interval [79], so that here the first set overrides the second. In this example one can see that the first set of classes, applied by Ban to the bass as well as the melody, produces the same unfortunate harmonic consequences as his theory of accents, namely, constant parallel motion between treble and bass.

But rather than trying in vain to put Ban's system of melodic intervals in order, I shall do better to point out that there is a considerable measure of agreement between him and his critics on this matter, as we have seen there was on the question of modes and major and minor consonances. His critics assumed, as he did, that semitones and minor thirds are soft and sad, whether rising or falling, and (though there is less evidence for this, owing to their interpretation of the poem) that major sixths, fifths and so on are vehement and energetic; they also agreed that falling intervals are appropriate to weakness and dying, and rising ones to energy and protest. Descartes, for example, admired in Boësset's air the

[78] Mersenne, *Corresp.*, X, 37-8.

[79] Ibid., X, 33. Ban has forgotten that he criticized Boësset's soprano at "mourir" for not rising at an acute accent.

rising semitone at "voir" and falling intervals at "mourir" as peculiarly suitable to a lover readily, but sadly, offering up his life to his mistress, while he considered the rising major sixth at "trop aymable" as equally suitable to the sudden elevation of the lover's spirits at the thought of the beautiful object of his affections [80]. Mersenne, in the *Harmonie Universelle*, had reproduced, in a bowdlerized form, Kepler's remarkable sexual explanation of the difference between the affective qualities of major and minor thirds as melodic intervals [81]. Nothing so original, or so convincing, however, appeared in Mersenne's or anyone else's contribution to the competition controversy on the subject of melodic intervals. But on other matters Descartes' long letter to Ban defending Boësset's air is so much more intelligent and sensitive than any of the other contributions that it is worth discussing it in some detail.

Descartes, in a letter to Mersenne of December 1640, stated that he had no doubt at all that Boësset's air was immeasurably better than Ban's [82]; the latter's air was like a schoolboy's exercise in rhetoric, carefully obeying all the rules, compared with an oration of Cicero, in which one is not even aware of there being any rules. But he went on to say that he had nevertheless a very high opinion of Ban both as a friend and as a musician, and also that he approved of rules, in music as much as in rhetoric.

In his letter to Ban Descartes was more tactful and less straightforward [83]. He begins by excusing himself for presuming to give an opinion on the two airs—he who is unable to sing a consonance or judge one by ear (referring presumably to his deafness); he is defending Boësset's air against Ban's criticisms just because it is natural for him, a Frenchman, to support a French composer. And he ends the letter by disclaiming any serious intentions in what he has written. His arguments, depending on the interpre-

[80] Descartes & Huygens, *Corresp.*, p. 294.

[81] Mersenne, *Harm. Univ.*, des Genres, p. 188; cf. supra p. 67.

[82] And that Ban now admitted this; which I find difficult to believe; Mersenne, *Corresp.*, X, 325-6.

[83] Descartes & Huygens, *Corresp.*, pp. 294, 298.

tation of a trivial love-poem, have rested on reasons that are "moral" (i.e. psychological) rather than mathematical or physical, and he could use the same reasons to argue the opposite case. We should not, I think, take too seriously these modest disclaimers of competence and earnestness. He had certainly examined Boësset's air with great attention, and his defence of it shows remarkable insight and subtlety.

Like the other French critics, Descartes finds in the text, not indignation and angry protest, but "only the softest emotions of love, of despair, sadness and obedience" [34]. But he limits these emotions to the first four lines, and argues that there is a change of feeling in the last two lines, a change that the poet has marked by the shorter, penultimate line—Descartes is the only person involved in the competition to note that this line is octosyllabic. The change is from total submission on the part of the lover to his cold mistress, from a resigned acceptance of death, to an incipient hope of posthumous revenge for his sufferings: "for he wants his mistress to remember that he has died miserable and lived faithful, hoping that it will come to pass that she will feel remorse for her cruelty and be tortured by missing him" [35]. Descartes also claims that this change of tone is reflected in the music.

Before coming to his demonstration of this change, I want to point out some examples, in his examination of the first section of the air (the first four lines), of his careful observation and musical sensitivity. I have already mentioned his and Mersenne's appreciation of the falling semitone for "amour"; but Descartes also notes that the setting of "moment" is related to that of "amour", being in fact the same melodically and harmonically [36]:

[34] Ibid., p. 294: "hic nullam plane indignationem, nec iram, sed blandissimos tantum amoris, abiectionis, tristitiae, et obedientiae affectus".

[35] Ibid., p. 295: "Quantum autem ad ultimos duos versus, notandum in iis aliquo modo mutari sententiam priorum; postquam enim amans summam obedientiam testatus est, hic de ultione nonnulla incipit cogitare; vult enim ut amica recordetur, se mori miserum et vixisse fidelem, sperans fore ut ipsam postea crudelitatis suae poeniteat eiusque desiderio torqueatur".

[36] Ibid.; "Vox etiam *moment* ad vocem *amour* relata, eodem quo illa semitonio recte mollitur; atque in eo est pathos quod amans iamiam et sine mora ut amicae placeat mori velit".

> The word *moment* being related to the word *amour* is rightly softened by the same semitone as the latter; and in this there is *pathos* in that the lover straight away and without delay wishes to die, that he may please his mistress.

He notices a similar musical identity in the setting of two words related both by meaning and sound, "plaisir" and "desir", the last syllables of which rise melodically "cum speciali gratiâ" by a tone, and have the same Phrygian cadence [37]. In neither case does Descartes mention the harmonic identity, but it is unlikely that he was unaware of it; for he points out the harmonic appropriateness of the interrupted cadence on to D minor for "haine", though he is slightly worried that the minor tenth, D-f, of the second chord might be considered "bland" rather than "sad" [38].

With regard to the change of mood in the last two lines, Descartes first argues that Boësset has expressed it by a change to triple rhythm, "for revenge requires a much more excited movement than does very sad obedience" [39]. This argument is at first sight puzzling, because in the version given by Ban no such change is marked, and most of the first section appears also to be in triple rhythm. But Ban as well notes the change (with disapproval), and the solution to the puzzle must be that both of them read the rhythm of the first section as, in modern terms, $\frac{2}{2}$ time, and that of the second section as $\frac{3}{2}$. This solution is borne out by the (not very consistent) barring of the two sections. The other musical expression of the change of mood between the two sections of the poem consists in the different kinds of cadence Boësset has used for the ends of lines or other points of repose in the verse. In the first section these are melodically falling, with the exceptions, already mentioned, of "plaisir" and "desir", and the whole movement of every phrase is downwards towards the cadence; whereas in the second section the reverse is the case, the

[37] Ibid., pp. 295-6.

[38] Ibid., p. 297.

[39] Ibid., p. 295 :"Quae mutatio affectûs clarissime a Poeta per quinti versus abbreviationem, et artificiosissime a Musurgo per triplam mensuram, expressa est: ultio enim multò concitatiorem motum quam tristissima obedientia requirit".

cadences rising melodically at "souviens toy", "cruelle" and "malheureux" (twice), and the melodic movement being always upward to the cadences. The downward movement is said to be suitable to the despairing obedience of the first section of the poem, and the upward movement to the threat of posthumous revenge of the second section [90]. These arguments in favour of a change of mood both in the poem and in Boësset's setting seem to me convincing. Even if one was unaware of the change before Descartes pointed it out, as I was, once he has done so, one can see that it is really there. This is a critical achievement of a high order.

Descartes' ideas on polyphony in general, and in particular in Boësset's air, are equally interesting and ingenious, but much less convincing. He is arguing against Ban's demand for rigidly syllabic homophony, and begins by pointing out that intelligibility of text could much more simply be achieved by using monody. Since we do not in fact always sing monodically [91],

> this is an indication that something else is sought for in polyphony than the easy perception of the words, namely, we look for the expression of the different emotions which the same words can arouse in different men, and also for pleasure from variety.

To achieve this end, one needs not only sounds of different pitch in different voices, which homophony allows, but also diversity of rhythms, which it does not. In a properly polyphonic piece, then, with some real independence of voices, each listener will pay greatest attention to the voice which he feels best expresses

[90] Ibid., pp. 295-6: "Praecipuum etiam artificium in eo est quod priorum quatuor versuum membra omnia in grauem sonum desinant (eo tantum excepto in quo est verbum *plaisir*, eiusque coniugati verbi *desir* syllabâ ultimâ, quae cum speciali gratiâ nonnihil eleuatur) et membra omnia duorum posteriorum desinant in acutum; quia nempe ut obedientis vox deprimi debebat, ita postea monentis ut meminerit, et quamdam ultionem quasi minantis, debuit attolli".

[91] Ibid.: "quod cum non fiat, indicium est aliud quaeri ex concentu quam facilitatem perceptionis verborum: nempe quaeritur expressio diuersorum affectuum qui ab iisdem verbis in diuersis hominibus possunt excitari, simulque ex varietate delectatio. Cui fini non sola diuersitas sonorum, sed etiam numerorum et temporum est accomodata. Cum autem plures audiunt eundem concentum, ad eam quisque vocem maxime attendit a qua suum affectum melius exprimi sentit atque ab illa praecipue mouetur".

his own emotional reaction to the text and will be moved chiefly by that voice. In Boësset's air Descartes considers that the bass is less humbly obedient than the treble, is more querulous, protesting and impatient; this is shown by its beginning earlier at "me veux tu", "viens donner" and "l'excez de mon amour", and its repeating the latter two phrases. Descartes also notes the frequent contrary motion between bass and treble, which he takes to indicate the expression of different emotions. His theory, therefore, unlike Ban's, does not lead to the unfortunate harmonic consequence of parallel motion between treble and bass.

Descartes also provides an alternative justification of polyphony [92]:

> Since music ought to imitate everything that happens in ordinary life, and often in fights and tumults several people say different things at the same time, why should we not allow music to imitate even this confusion?

He realizes, however, that music cannot always be representing scenes of tumult and confusion, and that, in any case, this justification is not applicable to Boësset's air. In giving this justification of polyphony Descartes may have had actual compositions in mind, such as Vecchi's madrigals or Jannequin's programmatic *chansons*. As far as I know, this could not have been the case for his other justification—that the different voices of a polyphonic piece express different emotional aspects of the same text; he was living many years before the great ensembles of Mozart's or Verdi's operas, which in any case usually involve the simultaneous singing of different texts. But he was correct, and I think original, in stating that polyphonic music can simultaneously express various distinct, and even contrasting emotions.

The main historical interest of the competition, however, concerns the fundamental differences in point of view between

[92] Ibid., pp. 296-7: "Praeterea, cum eorum omnium quae in communi vita accidunt musica esse debeat imitatrix, saepe autem in rixis et tumultibus plures eodem tempore diuersa loquantur, quare non etiam confusionis istius imitationem ei concedimus?".

Ban and his opponents. Ban, as we have seen, in spite of his claim to be original and his contempt for the ancients, took over the basic principles of extreme musical humanism: the production of specific emotional effects as the chief, even the sole function of music, and the resultant total subordination of music to text—vocal music is assimilated to oratory and becomes no more than an emotionally heightened kind of speech. It is significant that the only modern composer Ban admired was Monteverdi; for a similar humanist aesthetic, though less extreme and crude than Ban's, lies behind early Italian opera and the *stilo rappresentativo*, and his ideas would probably have seemed less strange and unacceptable in Italy than they were in France. That they were unacceptable in France is evident, and they were frontally attacked by Mersenne in his detailed examination, addressed to Huygens, of Ban's comments on the two airs. Huygens passed the letter on to Ban, who underlined some passages, and added some angry, but unenlightening marginalia. Mersenne starts off[93]:

> Now, to begin this examination, we must first suppose that music, and consequently airs are composed particularly *and principally to charm the mind and the ear* and to enable us to spend our lives with a little sweetness amidst all the bitterness we meet with.

Having laid down this basic principle (which Ban has underlined) that the function of music is to give pleasure, both intellectual

[93] Mersenne, *Corresp.*, X, 237-8: "Or pour commencer cet examen, il faut premierement supposer que la musique, et par consequent les airs, sont faictz particulierement *et principalement pour charmer l'esprit et l'oreille* et pour nous faire passer la vie avec un peu de douceur parmy les amertumes qui s'y rencontrent. Car de s'imaginer que la musique serve pour nous persuader le dessein du musicien aussy parfaictement comme feroit un bon orateur, et qu'elle ayt une esgalle force pour conduire à la vertu et pour faire hayr le vice que la voix d'un bon predicateur, bien qu'on chantast les mesmes choses qu'il recite en chaire, et de croire qu'en chantant l'on puisse aussy aisement instruire qu'en parlant et en discourant, c'est ce qu'il est difficile de se persuader, si l'on n'en veoit premierement l'experience.

Semblablement les airs *ne se font pas pour exciter la cholere, et plusieurs aultres passions*, mais pour resjouyr l'esprit des auditeurs, et quelquefois pour les porter à la devotion comme il arrive aux recits que l'on faict dans les Eglises durant le service divin. Je ne veux pas nyer que certains airs bien faicts selon la lettre, n'esmeuvent à la pityé, à la compassion, au regret, et à d'aultres passions, mais seulement que ce n'est pas là leur but principal, mais de resjouyr, ou mesme de remplir les sçavans auditeurs d'admiration, qui leur faict rechercher les causes d'un effect si signalé".

and sensual, Mersenne goes on to question even the practical possibility of Ban's assimilation of music to oratory:

> For to imagine that music is used to persuade us into accepting the musician's intentions as successfully as a good orator could do, and that music can lead us to virtue and make us hate vice with a force equal to the voice of a good preacher, even if one sang the same things that he spoke in the pulpit[94], and to believe that one can give instruction as easily by singing as by talking and lecturing, this is what is difficult to accept, unless one has first seen the experiment carried out.

Then comes a carefully qualified statement on the doctrine of the effects and the relation between music and text:

> Similarly, airs *are not made in order to excite anger and several other passions*, but to give enjoyment to the minds of the listeners, and sometimes to incline them to religious devotion, as in church music. I do not wish to deny that certain airs which are well composed with regard to the text and its meaning may move to pity, compassion, regret and other passions, but only that this is not their chief aim, which is to give enjoyment, and even to fill the learnèd listeners with admiration and lead them to search for the cause of such a remarkable effect.

That is to say, Mersenne does not deny that the emotional effects exist and that they are produced by the musical setting being rightly fitted to the text; but he keeps the effects within the bounds of his basic aesthetic principle—the effects are just one factor in the pleasure afforded by a beautiful song.

It is not, I think, by chance that he chooses anger as an example of the effects when he is belittling their importance, and milder emotions, such as religious devotion, pity and so forth, when he is admitting their part in the excellence of a song. It is always a weak spot in any aesthetic positing the production by a work of art of specific emotional effects on the beholder or listener that in many cases these emotions are by general admission unpleasant, and that nevertheless people appear to enjoy seeing or hearing

[94] Ban was particularly interested in the reform of church music; see Mersenne, *Corresp.*, VIII, 81-3 (Ban to Mersenne), 342-3 (Ban to Doni).

works that portray such emotions. It is perhaps just possible to claim that the spectator of a tragedy enjoys the process of being purged by pity and terror, or at least the state of tranquil purgation at the end of the performance; but it is very difficult to believe that anyone enjoys being in a state of frustrated anger. One reason, of course, why Mersenne chose anger as his first example was that Ban intended his air to express angry indignation. But he may also have had in mind the story, which he certainly knew, of a young man's being aroused to martial frenzy by Claude Le Jeune's music at the Duc de Joyeuse's marriage in 1581, as long before Alexander had been by Timotheus [95]. This would account for the curious argument at the end of the above passage: the young man may not have enjoyed being in a state of furious rage, but the learnèd spectators enjoyed the occasion offered to discuss the marvellous effects of antique music.

Mersenne was aware perhaps that one source of the basic disagreements between Ban and himself and the other French musicians was that Ban was influenced by Italian musical theory and practice. Ban had proposed altering Boësset's setting of "cruelle" to one with a leap of a fifth on to the second syllable [96]:

Ex. 4

on which Mersenne comments [97]:

> this exclamation would perhaps be accepted in Italy, whence he [Ban] seems to have learnt this harshness and violence which is found in the airs sung there; but . . . the French represent their passions with less violence . . .

[95] The story was told to Mersenne by Titelouze in a letter of 1622 (Mersenne, *Corresp.*, I, 75); cf. F. A. Yates, "Poésie et musique dans les 'Magnificences' au mariage du duc de Joyeuse, Paris, 1581", *Musique et Poésie au XVIe siècle* (Colloque du C.N.R.S.), Paris, 1954, pp. 243-4.

[96] Mersenne, *Corresp.*, X, 28.

[97] Ibid., X, 247-8: "cette exclamation seroit peut-estre reçüe en Italie, d'où il semble avoir apris ces duretez et ces violences dans les airs que l'on y chante; mais . . . les François representent leurs passions avec moins de violence".

In fact Ban's air is more like a very poor imitation of an Italian operatic aria than it is like a French *air de cour*.

The other fundamental difference in outlook between Ban and the French arose from the former's assumption that one could construct a system of rules for composition by deduction from a few axiomatic first principles. This assumption was of course the main source, apart from his natural arrogance, of all Ban's errors and absurdities, and also of his originality; he had tried to cut himself off from the musical practice of his time and to work out his rules without reference to that practice. Here again Mersenne shows clearly that he is aware of the basic difference of method between Ban and himself. According to Mersenne, a theory of composition should be constructed, not by deduction from first principles, but by induction from the practice of great composers. He ends his long letter to Huygens about Ban with the following suggestion [98]:

> perhaps someone will undertake the task of making laws and rules for beautiful songs from the songs of our Orpheus [i.e. Boësset], so that, as he who comes nearest to Cicero's style is considered to be the most elegant author, so the composers who imitate the most perfectly the method he uses to compose his airs will be judged the most excellent.

It remains only for me sadly to record that, although national prejudice certainly played its part, it was not only the French who considered Ban mistaken in his theory of composition, hopelessly inept as a composer, and irremediably self-satisfied. In January 1641 Huygens wrote to Boësset, saying that Ban was extremely learnèd in the mathematical part of musical theory [99]:

[98] Ibid., X, 249: "quelques-uns entreprendront peut-estre de faire des loix et des reigles des beaux chants sur ceux de nostre Orphée, afin que comme celuy qui aproche le plus près du stile de Ciceron, est estimé composer le plus elegamment, de mesme les compositeurs qui imiteront plus parfaictement la methode dont il use pour faire ses airs, soient jugez les plus excellentz".

[99] Ibid., X, 416-7: "Mais pour ce qui est de l'application de l'art, et nommement de ce vray genie qui ne s'enseigne à personne et qui fait l'ame de la pratique, il y entend aussi peu, que vous, Monsieur, en possedez amplement et au ravissement de tout le monde. Les regles d'ailleurs qu'il pretend de prescrire au compositeur d'un air à l'advenant de la lettre, sont à mon advis si esloignées de raison que, quand je

> But, as far as the application of the art is concerned, and in particular that true genius which cannot be taught and which is the soul of practical music, he knows as little about it as you, Sir, are fully a master of it, to the admiration of everyone. Moreover, the rules which he presumes to prescribe to composers of airs for setting the text are in my opinion so unreasonable that, even if I had not seen the poor attempt [i.e. Ban's air] he has sent you, I should, like you, have rejected them.

Mersenne had proposed, in case Ban should not accept the verdict of only French experts, to extend the competition by sending the two airs to all the royal courts of Europe [100]; referring to this proposal, Huygens continues [101]:

> It will be fun to see the verdicts of the best musicians of Europe on this, to which he [Ban] is willing to submit; but, condemned as he will be, he will hang on to his own opinion, if I know the man.

n'auroy pas veu le mauvais essay qu'il vous en a envoyé, je ne lairrois pas de les rejeter avecq vous".

[100] Ibid., X, 206-7, 412.

[101] Ibid., X, 417: "Il y aura du plaisir à veoir là-dessus les arbitrages des meilleurs musiciens de l'Europe, auxquels il est content de s'en remettre: mais, tout condamné qu'il sera, il ne demordra jamais de son imagination, si je le cognoy".

CHAPTER VII

SEVENTEENTH–CENTURY SCIENTISTS' VIEWS ON INTONATION

In this chapter I am taking the term "scientist" in a broad sense, and shall include the views of some thinkers who were more musicians than scientists in the modern sense of the word. After all, at this period music was still a branch of mathematics, along with the other three liberal arts, geometry, arithmetic and astronomy. But mostly I shall be considering the musical theories of thinkers who were also eminent in other fields. These are too well known to need any introduction: Simon Stevin, Kepler, Isaac Beeckman, Descartes, Mersenne, Francis Bacon, Galileo Galilei, Lord Brouncker, John Wallis, Christiaan Huygens.

These scientists, when considering systems of intonation, were faced with the following possibilities:

1) The Pythagorean scale.
2) Just intonation, either absolutely just and hence extremely unstable, or Ptolemy's scale with its narrow fifth and minor third.
3) Some kind of temperament, of which there were two main classes: first, some kind of mean-tone temperament; second, equal temperament.

Among these possibilities what were the choices in fact made by scientists and musical theorists of our period?

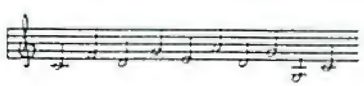
1) The Pythagorean scale became obsolete after Zarlino began publishing in the 1550's, except for a few extreme humanists, such as Girolamo Mei and, following him, Vincenzo Galilei in his *Dialogo* (but only in this work) [1], and for archaic, ill-informed authors, such as Robert Fludd [2].

[1] See Walker, "Musical Humanism", ii, 121.

[2] V. supra Ch. I, p. 2, n. 6.

2) Just intonation was accepted by the great majority as the best and most natural system; but there was an important dissenting minority, which I will deal with later. The problem of the instability of absolutely just intonation, or, in Ptolemy's scale, the great difficulty of teaching singers to sing both just fifths and one slightly narrow one, were usually evaded, as we have seen in the case of Vincenzo Galilei and Zarlino [3]. Some theorists, such as Ban, like others before and after him, suggested using a keyboard with extra keys by which one could get very near to just intonation [4].

Christiaan Huygens is, I think, unique in both tackling the problem and suggesting a plausible solution. He begins by denying Zarlino's and Galilei's assumption that unaccompanied singers use absolutely just intonation. He believes that they probably deviate more from just intonation than do tempered instruments. But, even if they did sing entirely just intervals, they would inevitably lose or gain pitch in the course of a song. In fact, says Huygens, one does often find that singers have risen or fallen by a semitone or more at the end of a piece. He suggests that the reason why a well-trained singer does not do this is because his memory is good enough to preserve the pitch of the original starting-point. He proposes the following experiment to demonstrate this point. If you sing slowly this tune:



and succeed in singing entirely just consonances, you will end up two commas flat ($(81:80)^2$). But, if it is sung quickly, says Huygens, "je trouve que le souvenir de ce premier ut retient la voix dans le ton", and that therefore some of the consonances must have been tempered [5].

In spite of this, Huygens, like Zarlino, Galilei father and son, Kepler, Descartes and Wallis, has no doubt that just intonation

[3] V. supra p. 17.
[4] V. supra p. 91.
[5] Christiaan Huygens, *Oeuvres Complètes*, XX, 76-7, 101.

gives the most perfect consonances. Writing in about 1661 against Stevin (whom I will discuss at the end of this chapter), he states [6]:

> It is established by experience, and those who have even the slightest ear for music cannot deny, that consonances in the above proportions [he has just given the just ratios] are very perfect and better than when one deviates from these true numerical proportions. And those who have dared to maintain the contrary and that the fifth does not consist in the ratio 3 to 2, either did not have an ear capable of judging it, or thought they had a reason for their opinion, but their conclusion was false.

The "experience" here mentioned almost certainly means experiments using a monochord, the κανων μονοχορδος, that ancient scientific instrument, described in detail by Ptolemy, which consists of a string stretched over a ruler with a moveable bridge. If the instrument is fairly long and the string fairly true, it can give quite convincing empirical proof that the consonances are in the simple ratios 2:1, 3:2, etc.. We know that Huygens' monochord was five foot long [7]. But this kind of empirical proof is of course only valid if it is true that everyone not tone-deaf at all periods everywhere has always agreed on what a perfect fifth sounds like. The elder Galilei made the disquieting observation that many musicians he knew, having constantly heard tempered fifths, had come to prefer slightly narrow fifths to just ones [8].

Another, subsidiary reason for preferring just intonation, as opposed to any tempered system, was that some thinkers hoped thereby to achieve again the marvellous moral and emotional effects of ancient music. In 1688 Thomas Salmon, a clergyman, published *A Proposal to Perform Musick in Perfect and Mathematical Proportions*. From the title-page of this book we learn that it was

[6] Ibid., XX, 32, 169: "Il est constant par l'experience, et ceux qui ont tant soit peu d'oreille pour la musique ne peuvent nier, que les consonances suivant les proportions susdites ne soient très parfaites et meilleures que quand on s'ecarte de ces veritables proportions numeriques. Et ceux qui ont osé soustenir le contraire et que la quinte ne consistast pas dans la raison de 3 a 2, ou n'avoient pas l'oreille capable d'en juger ou croioient avoir une raison pour cela, mais ils concluoient mal".
[7] Ibid., XX, 47. Cf. letter of Gabriel de la Charlonye to Mersenne of June 1633 (Mersenne, *Corresp.*, III, 413-6) expressing doubts on the accuracy of the monochord.
[8] V. supra p. 22.

"Approved by both the Mathematick Professors of the University of Oxford", and is supplied with "large Remarks upon this whole Treatise, by the Reverend and Learned John Wallis, D.D.". Wallis offered these remarks to Salmon not "by way of contradiction to your design (which I approve of)", but to clarify certain points [9].

Salmon begins by noting the recent revival of interest in ancient music shown by the publication of editions of ancient authors hitherto available only in manuscript or bad translations, such as Meibom's edition in 1652 of Aristoxenos, Aristides Quintilianus, and others [10], and Wallis's edition and translation of Ptolemy's *Harmonica* in 1682 [11]. Judging from these treatises, Salmon, like the majority of seventeenth-century scholars, considers that the Greeks did not have such "a pleasing sweetness of Air, such a various contexture of Chords" as we find in modern music, i.e. they had no polyphony. It might therefore seem that

> the mighty Power of Musick, Recorded by the most Grave and Authentick Historians, may be lookt upon as Romance, since all the Excellencies now perform'd, cannot conquer the Soul, and subdue the Passions as has been done of Old.

But before we jump to this conclusion let us inquire whether there may not be "something very Considerable still wanting, something Fundamental very much amiss", namely, "the Accurate Observation of Proportions which the Soul is from Heaven inform'd to Judge of, and the Body in Union with it, must Submit to". Isaac Vossius, in his *De Poematum Cantu & viribus Rhythmi* (1673), has already demonstrated the importance of the preservation of poetic metre and the intelligibility of text for exciting or soothing the passions; but the ratios of musical intervals may be equally essential for achieving the Effects:

> Till both the Fundamental Points be observed with such Exactness & Excellency as the Ancients took care of; we must not say we do all they did, or that they could not prevail more than we can [12].

9 Salmon, *A Proposal*, p. 29.
10 Marcus Meibomius, *Antiquae Musicae scriptores septem*, 1652.
11 *Claudii Ptolomei Harmonicorum libri Tres*, ed. John Wallis, Oxford, 1682.
12 Salmon, ibid., pp. 1-4.

Salmon then lists the just ratios for the major and minor scales [13], and gives as his authorities for proposing scales that contain unequal tones: Descartes, Gassendi, Wallis,

> and all other Learned men, who have in this last Age reviewed the Harmonical concerns. 'Tis time certainly to receive into practice those Improvements, which the greatest Modern Philosophers in the World have afforded Musick [14].

The practical difficulties of employing such scales on fretted instruments, one of the main obstacles to their use, are dealt with by Salmon in a rather off-hand manner. He realizes that each string will have to have its own frets, and that all these will have to be changed for every key; he suggests that one might have a set of different, changeable finger-boards, and finally leaves these problems to "the Mechanick" [15].

A similar attempt to construct fretted and keyboard instruments that could be played in just intonation, and also in the ancient chromatic and enharmonic genera, was made in the 1630's and 1640's by Giambattista Doni, a friend of Galilei and correspondent of Mersenne. His main motive also was to revive the marvellous effects of ancient music; but, unlike Salmon, Wallis and the majority of scholars, he believed that ancient music was polyphonic [16].

Among all these advocates of just intonation Kepler alone *both* fully realized the historical connexion between the rise of polyphony in late mediaeval and modern times and the change from Pythagorean to just intonation, i.e. from dissonant thirds and sixths to consonant ones, *and* saw the invention of polyphony as the extraordinary and unique advance that it was.

With regard to our third possibility, some kind of temperament, it was generally admitted that this was actually used and was

13 Ibid., pp. 9-12; he gives the major and minor tones in a different order from Ptolemy's scale; this is corrected by Wallis (ibid., p. 37).
14 Ibid., p. 12.
15 Ibid., pp. 17-8; cf. p. 41 for Wallis on this.
16 See letters from Doni to Mersenne, Mersenne, *Corresp.*, III, 508-9; V, 311; IX, 218-21, 486-90. Cf. Walker, "Musical Humanism", iii, 57-60.

inevitable on instruments of fixed intonation, although, as we have seen with Ban, Salmon and Doni, there were attempts to avoid or mitigate temperament by multiplying the number of keys or frets, and these attempts go back to the mid-sixteenth century. For keyboard instruments some kind of mean-tone temperament was considered to be the only tolerable tuning, superior to equal temperament, with its harsher thirds and sixths [17]; but equal temperament was the only practicable possibility for fretted instruments. From the second half of the sixteenth century onwards musical theorists offered various solutions to the problems presented by the irrational quantities involved in temperament. Galilei, as we have seen, gave for equal temperament a simple method of approximating to $\sqrt[12]{2}$, namely, by taking away successively an eighteenth part of the string-length. The error, which Kepler calculated, is very small: $(\frac{17}{18})^{12} = .50363$ [18]. There were also quasi-geometric methods, which Zarlino gives, of finding two mean proportionals between 1 and 2, thus giving the cube root of 2; then, taking twice the square-root of this quantity, one gets the twelfth root. The elder Galilei knew of the solid geometric construction of Archytas for finding two mean proportionals, though this of course is not much use for the practical job of fretting the neck of a lute [19]. As I have already mentioned, logarithms were not used for this purpose until surprisingly late. And this was not from a lack of interest in temperament—Mersenne, for instance, gives several approximations to equal temperament by various mathematicians, but none based on the simple procedure of dividing log 2 by 12 [20].

[17] On the unpleasantness of equally tempered key-board instruments, see G. B. Doni, *Annotazioni*, pp. 303-4.

[18] V. Galilei, *Dialogo*, pp. 49-53 (cf. *Discorso*, p. 55); Kepler, *Ges. Werke*, VI, 143-5.

[19] V. Galilei, *Discorso*, p. 68; cf. Barbour, op. cit., pp. 49-55; Sir Thomas Heath, *A History of Greek Mathematics*, Oxford, 1921, pp. 246-9; Mersenne, *Corresp.*, I, 206, 257.

[20] On Mersenne and equal temperament, see Hellmut Ludwig, op. cit., pp. 72-5 (he exaggerates Mersenne's commitment to equal temperament); Mersenne, *Corresp.*, III, 418-9; IV, 52, 435-7; V, 69-73; *Harm. Univ.*, Nouvelles Observations (at end), p. 19 (on the disadvantages of equal temperament).

The first person, as far as I know, to use logarithms to divide musical intervals into equal steps was the translator into English and annotator of Descartes' *Compendium Musicae*, which appeared in 1653 [21]. This work is usually ascribed to Lord Brouncker, later President of the Royal Society [22]. And here is a surprising fact, which I find totally baffling. Instead of dividing an octave into twelve equal semitones, Brouncker divides an interval approximating to an eleventh (octave + 4th) into seventeen equal steps. He obtains this interval by dividing his string-length in extreme and mean proportion, i.e. the Golden Section, thus:

so that B:A = A:B-A. The ratio of B-A:B is then approximately an eleventh; this you can see if you remember that, in the Fibonacci numbers, an approximation to the Golden Section is 3, 5, 8, and 3:8 is the ratio of a just eleventh. Brouncker then, taking B = 10, extracts by logarithms the seventeenth root of B:B-A, and lists his seventeen semitones, both in logs and in numbers, starting with the highest note, B-A, until he has reached to lowest, the open string B. Since this interval is slightly wider than both a just and an equally tempered eleventh, Brouncker's system contains no true octaves, a defect which makes it quite useless for musical purposes. He gives no explanation whatever for this extraordinarily crazy procedure, and I have been able to find no comment on it, either contemporary or modern [23].

The only explanation that I can suggest, which is far from satisfactory, is that the Golden Section had acquired such a numinous quality that it befuddled the minds of otherwise quite sane and competent mathematicians. Remember, for example, Luca Pacioli's *Divina Proportione* [24], or Kepler's poetic and il-

[21] Renatus Des-cartes, *Excellent Compendium of Musick*: with Necessary and Judicious Animadversions Thereupon. By a Person of Honour, London, 1653.

[22] See *Dictionary of Scientific Biography*, New York, Vol II, 1970, pp. 506-7, art. Brouncker, by John Dubbey.

[23] Descartes, *Compendium*, ed. cit., pp. 84-6.

[24] Luca Pacioli, *De Divina Proportione*, Milan, 1956 (Fontes Ambrosiani, XXXI).

luminating use of the Golden Section and the Fibonacci numbers in his musical theory [25]; and remember that, for both these thinkers, the Golden Section is closely associated with the regular polygons and the regular solids, which in Plato's *Timaeus* and Kepler's astronomy are basic, formative constituents of the universe. None of these thinkers of course were either befuddled or crazy; but one can see that in their systems the Golden Section is beginning to take on almost supernatural, divine powers.

Another example of the use of the Golden Section for equal temperament that is, in its way, as crazy as Brouncker's is that of Salinas. In his *De Musica* (1577), after having accurately described equal temperament and advised the use of the mesolabe to find eleven mean proportionals between 1 and $\frac{1}{2}$, he adds another "recently discovered method" ("modum nuper quidem excogitatum"), which is to construct, as in Euclid VI, 30, a line divided according to the Golden Section:

so that ab:ae = ae:eb. Then divide eb in the same way at f, and fb at g, and so on, twelve times. If ab is a string and twelve frets are placed at these points, then, Salinas claims, they will make equal semitones, and the highest fret at e will make an octave with the whole string. Since e clearly does not divide ab in half, and the spaces between the frets get bigger instead of smaller, one must suppose that something has gone wrong with the diagram, and remember that Salinas was blind, as Mersenne kindly suggested. But, even if one emends the diagram so as to correct these two patent errors, there is no conceivable way in which the ratio of the Golden Section, $\frac{1}{2}(\sqrt{5}-1):1$, could lead one to the ratio $(\frac{1}{2})^{1/12}:1$. Huygens' comment on this passage of Salinas is short and correct: "paralogisme" [26].

[25] V. supra p. 51 seq.
[26] Salinas, *De Musica*, III, xxxi, pp. 173-4; Huygens, *Oeuvres*, XX, 114; Mersenne, *Harm. Univ.*, IV des Instr., pp. 224-5. Cf. Beeckman's mad geometrical construction for equal temperament in his *Journal*, ed. C. de Waard, La Haye, 1939, I, 180-1.

After Brouncker, the only other people I know of who used logarithms for musical purposes are Newton and Huygens. Huygens divided the octave into thirty-one equal steps, in order to achieve, not equal temperament, but a close approximation to mean-tone. His work was not published until 1691 [27].

I now come to the small minority who did not accept just intonation. First, there are a few writers on music who abandon the whole mathematical tradition, such as Francis Bacon and Campanella [28]. The second Century of Bacon's *Sylva Sylvarum* (1627) consists of experiments concerning music and sound. It begins [29]:

> Music, in the practice, hath been well pursued, and in good variety; but in the theory, and especially in the yielding of the causes of the pratique, very weakly; being reduced into certain mystical subtilities, of no use and not much truth.

These mystical subtleties are presumably the theory of musical ratios, discovered in pre-Platonic times, empirically established by the monochord, and elaborated and adapted to polyphonic music by sixteenth-century theorists. In fact, Bacon never mentions either the ratios or the monochord. When dealing with the octave and the likeness of this consonance to unison, he remarks: "The cause is dark, and hath not been rendered by any" [30]. He compares what is "pleasing or ingrate to the hearing" with what is so to the sight, and concludes that in both cases what is pleasing is caused by "equality, good proportion, or correspondence". "But", he goes on, "to find the proportion of that correspondence is more abstruse" [31]. Later on he suggests experiments with different lengths and tensions of strings and lengths and diameters of pipes, in order to discover "the just and measured proportion of the air percussed, towards the baseness or trebleness of tones", "which", he says, "is one of the greatest secrets in the

[27] Huygens, *Oeuvres*, X, 169-74; XX, 6-7.
[28] On Campanella's musical theory, see Walker, *Magic*, pp. 230-4.
[29] Francis Bacon, *Works*, ed. Spedding, Ellis, etc., London, 1857-1901, II, 385.
[30] Ibid., II, 386.
[31] Ibid., II, 388.

contemplation of sounds" [32]. Since this dark cause and great secret had been common knowledge for two thousand years, Bacon's failure even to mention musical ratios seems to me surprising and perverse.

Far more interesting is Stevin's rejection of just intonation. This occurs in a short treatise on music in Dutch, which was not published until the nineteenth century [33]. But his views were quite widely known in the seventeenth century because his papers passed to Constantijn Huygens, and because his theory of intonation is briefly mentioned in one of his published works [34]. Beeckman, writing to Mersenne in 1629 about equal temperament, states that many years ago he had rejected Stevin's theory, after having for a time accepted it. Through Beeckman probably, Descartes knew of "l'erreur de Stevin" [35]; and, as I have mentioned, Christiaan Huygens wrote against him [36].

Stevin's treatise begins, *more geometrico*, with definitions and postulates. Natural singing is in the major diatonic scale, and in this scale all the tones are equal and so are all the semitones, which are exactly half a tone. He then gives the ratios of an equally tempered chromatic octave in string-lengths, going from 1 to $\frac{1}{2}$, using the notation $\surd^{(n)}$ for our $\sqrt[n]{\ }$, and avoiding fractional exponents by writing, e.g. $\surd^{(12)}$ 1/128 for $(1/2)^{7/12}$ [37]. Then follows a general discussion of ratios and proportions. The Dutch for ratio is *rede(n)*, and Stevin uses for proportion the word *everedenheijt*, i.e. equirationality. The relation, e.g., between the two ratios 6:3 and 8:4 is an *everedenheijt*. These Dutch (*duytsche*) words are simple and modest, but they are of infinite power. *Everedenheijt* immediately tells us its meaning; it is like "a definition of its substance" ("als bepaling sijns grondts"). The Greek and Latin terms, ἀναλογια and *proportio*, fail to do this. Hence came the mis-

[32] Ibid., II, 409-10.
[33] Simon Stevin, *The Principal Works*, ed. Ernst Crone, etc., Amsterdam, Vol. 5, 1966, pp. 413 seq., *Vande Spiegheling der Singkonst*, ed. A. D. Fokker.
[34] See Huygens, *Oeuvres*, XX, 32.
[35] Mersenne, *Corresp.*, II, 276, 286.
[36] V. supra p. 113.
[37] Stevin, op. cit., V, 422-7.

taken belief that 6, 4, 3, is a proportion, a belief which has produced "endless vanities" ("oneindelicke ydelheden") [38].

What Stevin is doing here is to object to the term *proportio* or *analogia* being used in a generic sense to mean several kinds of relation between ratios, in particular to include the harmonic one, 6, 4, 3, instead of being confined to what was in truth the original meaning of these terms, namely, the equivalence of two or more ratios, or geometric proportion. If only the Greeks had had a term like *everedenheijt* constantly before them, they would have stuck to geometric proportion and thus have arrived at equal temperament.

Not having the benefit of the Dutch language, which Stevin evidently thought to be the *Ursprache*, as did his compatriot and contemporary, Goropius Becanus [39], the Greeks, having discovered that the ratio of a fifth was very near ("seer naer") to 3:2, mistakenly took this to be the true ratio. From it they deduced the other intervals, ending with the semitone 256:243, i.e. the Pythagorean scale. "But experience shows patently to the hearing that this is not the semitone, because it is much too small". The Greeks ignored this evidence of their own ears, and even rejected the "sweet and lovely sounds" ("de soete ende lieflicke gheluyden") of thirds and sixths. Then Ptolemy introduced major and minor tones, which are unnatural, because in fact we sing all tones the same; and this system was further elaborated by Zarlino. All these mistakes occurred because the Greeks had no proper understanding of the concept of equirationality. And this was because they were without the Dutch language, which keeps this concept constantly before one's mind [40].

Stevin deals very shortly with the objection that so sweet a consonance as the fifth is unlikely to be based on such an "ineffable, irrational and inappropriate" number as $(\frac{1}{2})^{7/12}$.

[38] Ibid., V, 426-433.
[39] See Arno Borst, *Der Turmbau von Babel*, Stuttgart, 1957-63, pp. 1215 seq.
[40] Stevin, ibid., V, 431-3.

> Since it is not our intention to teach to the unspeakable irrationality and inappropriateness of such a misunderstanding of the speakability, rationality, appropriateness, and natural, wonderful perfection of these numbers, we shall leave it at that, because we have dealt with it elsewhere [41].

This is a reference to his *Arithmetique*, published in French in 1585, where he devotes five pages to arguing "Qu'il n'y a aucuns nombres absurdes, irrationels, irreguliers, inexplicables, ou sourds" [42]. Stevin here is really only objecting to the apparently pejorative quality of such terms, just as Kepler preferred to use the term "ineffable" rather than "irrational". But Stevin could, I think, have made out a strong case for his contention that equal temperament is more natural than systems which attempt to preserve the simple, rational ratios of the consonances and other melodic intervals. For these will just not fit tidily into a framework of octaves, whereas the equally tempered intervals do, and Nature plainly does not abhor irrational numbers, witness the diagonal of a square and π. To insist on preserving at all costs the ratios 3:2, 4:3, etc. was, from Stevin's point of view, like insisting that the diagonal of a square must be exactly 1.4 and π exactly 22/7 because these are rational numbers. But of course the insuperable objection to Stevin's position, the objection in fact made by his seventeenth-century critics, was the simple appeal to experience: a just major triad is undeniably sweeter than an equally tempered one. Or was Vincenzo Galilei right in stating that many people, even a majority, had come to prefer tempered consonances?

[41] Ibid., V, 440-1.
[42] Ibid., IIB, 532 seq.

CHAPTER VIII

THE MUSICAL THEORY OF GIUSEPPE TARTINI

i. Introduction *

Tartini was a remarkably independent and original musical theorist [1], but he achieved originality and independence at the cost of being archaic—a Renaissance thinker living and writing in the age of enlightenment. Moreover, he was fully aware of this; he apologizes defensively for using Aristotelian terminology [2], and there are bitter references to "this enlightened century" (*questo secolo Illuminato*) [3], which has not even tried to understand his system. In his first published treatise, the *Trattato di Musica secondo la vera scienza dell'armonia* (1754) [4], the only modern source he cites is Zarlino, and indeed there is no reason to suppose

* This chapter is based almost solely on Tartini's published writings. There is a large body of manuscript material which I know, if at all, only at second hand. My justification for nevertheless publishing this essay is: first, that I am too old to devote several years to examining this material, and, secondly, that since Tartini's ideas were known to his contemporaries and later theorists only through his published works an interpretation based mainly on these may help to illuminate his position in the history of musical theory.

The following two publications appeared too late to be used in this chapter: Leonardo Frasson, "Bibliografia Tartiniana", *Il Santo. Rivista Antoniana di Storia, Dottrina, Arte*, Padova, Serie II, Anno XVII, Fasc. 1-2, 1977, pp. 283-305; G. Tartini, *Scienza Platonica fondata nel cerchio*, a cura di Anna Todeschini Cavalla, Padova, 1977.

[1] The fullest modern work on Tartini is Antonio Capri, *Giuseppe Tartini*, Milano, 1945. Pierluigi Petrobelli has done valuable work on Tartini's life, *Giuseppe Tartini: Le Fonti biografiche*, Firenze, 1968, and published several articles on him, one of which, "Tartini, le sue idee e il suo tempo" (*Nuova Rivista Musicale Italiana*, Rome, 1967), covers important aspects of his thought omitted in my paper. A. Planchart's article, "A Study of the Theories of Giuseppe Tartini" (*Journal of Music Theory*, IV, 1960), is quite a full exposition.

[2] Tartini, *Trattato di Musica secondo la vera scienza dell'armonia*, Padova, 1754 (facsimile, 1973, by the Accademia Tartiniana di Padova), p. 31; *De' Principi dell'Armonia Musicale contenuta nel Diatonico Genere Dissertazione*, Padova, 1767 (facsimile, 1974, by the same), p. 113.

[3] Tartini, *Dissertazione*, pp. 37, 71-2.

[4] V. supra note 2. There is a useful German translation of, and commentary on this: Tartini, *Traktat über die Musik gemäss der wahren Wissenschaft von der Harmonie*, übersetzt und erläutert von Alfred Rubeli, Düsseldorf, 1966.

that he had read any later theorist [5], though some of Rameau's ideas must have filtered through to him, in particular the concept of a fundamental bass. By the time he published his second work, *De' Principi dell'Armonia Musicale contenuta nel Diatonico Genere Dissertazione* (1767), he was replying to French critics of the *Trattato*, Jean de Serre and d'Alembert [6], and was himself criticizing Rameau; but his system was already fixed, and this late reading of modern authors led to only minor adjustments. His reaction to Rameau, when he had read him, was one of unshaken confidence in his own system:

> Where M. Rameau has spoken the truth, he necessarily agrees with the truth of the present system. Where he has spoken falsely, he is necessarily detected by being confronted with the truth of the present system [7].

Rameau has the great merit of being "the first among the Professors to enter on the true path" [8]; but his system is basically defective, because founded on incomplete principles. Again, although at some period after the publication of the *Trattato* Tartini had at least heard of Kepler's *Harmonice Mundi* (1619) [9], I am sure that the striking likenesses between the musical theories of the astronomer and the violinist are not due to any direct influence, but to the fact that Tartini's mental world was that of the early seventeenth century.

But Tartini appears much less archaic when compared, not with his French contemporaries, but with his Italian ones, in

[5] Cf. letter from Tartini to Martini of 8 September 1752, referring to a mention by Martini of Mersenne: "Io faccio stato da me solo, e non ho lettura, ne eruditione de sorte alcuna; siche mi sono affatto incognite tutte le scoperte di tali uomini, e di questi tempi" (*Carteggio inedito del P. Giambattista Martini*, ed. F. Parisini, Bologna, 1888, p. 372).

[6] J. A. de Serre, *Observations sur les Principes de l'Harmonie*, Genève, 1763; J. d'Alembert, *Eléments de musique théorique et pratique*, Lyon, 1762 (cf. Capri, op. cit., p. 410); Tartini, *Risposta alla Critica di lui Trattato di Musica di Mons. Le Serre di Ginevra*, Venezia, 1767.

[7] Tartini, *Diss.*, p. 82: "dove M. Rameau ha detto il vero, s'incontra necessariamente col vero del presente sistema. Dove ha detto il falso, si scopre necessariamente a confronto del vero del presente sistema". Cf. Capri, op. cit., pp. 487-8.

[8] Tartini, ibid., "Il di lui merito è grande, perch'è stato il primo tra i Professori a calcar la vera via".

[9] Capri, op. cit., pp. 490, 493.

particular with two very learned musicians, Francescantonio Vallotti [10] and Giambattista Martini [11], with whom he was in close and frequent contact. Both these theorists had read Rameau carefully and understood him; but both rejected his system because of its physical basis, namely, the natural series of overtones and the determination of pitch by frequency of vibration rather than by string-length, and, with patriotic conservatism, they kept to the purely arithmetic basis of Zarlino [12].

The rejection by Tartini of frequency as the determinant of pitch is of extreme importance because it threatens the validity of the mathematical basis of his system. Since he thinks of musical ratios in terms of string-lengths instead of frequencies, the concept of musical harmony is for him indissolubly tied to that of harmonic proportion in the mathematical sense, whereas for those of his contemporaries who were thinking in terms of frequencies, harmonic proportions were transformed into arithmetic ones, for example, the ratios of the natural overtones, which, in string-lengths, give the harmonic series 1, $\frac{1}{2}$, $\frac{1}{3}$ . . . . , give, as frequencies, the arithmetic series 1, 2, 3 . . . . ; or the division of a fifth into the major triad, which for Zarlino and Tartini is harmonic

$$\overbrace{\underbrace{15 \quad 12}_{\text{maj. 3rd.}}\underbrace{\quad 10}_{\text{min. 3rd.}}}^{\text{fifth}}$$

becomes, in frequencies, arithmetic

$$\underbrace{\overbrace{4 \quad 5}^{\text{maj. 3rd.}}\overbrace{\quad 6}^{\text{min. 3rd.}}}_{\text{fifth}}$$

Since the whole mathematical basis of Tartini's system rests on a descending hierarchy of kinds of proportion: harmonic, arithmetic, geometric, the system loses its main contact with physical

[10] Cf. Capri, op. cit., pp. 42, 231-4; Rubeli, ed. cit. (supra note 4), pp. 11-2; and infra pp. 165-6.

[11] Cf. Rubeli, ed. cit., pp. 18-9, and supra note 5.

[12] See E. R. Jacobi, "Rameau and Padre Martini News Letters and Documents", *The Musical Quarterly*, New York, 1964, pp. 470-4.

realities once it is generally accepted, as it was, at least outside Italy, long before the middle of the eighteenth century, that frequency of vibration in the air is a more fundamental and general cause of musical pitch than length of string.

For theorists without such a hierarchy it was of no importance whether they dealt with ratios in string-lengths or in frequencies, since they are so easily convertible one into the other. Rameau, for example, sometimes gives ratios as string-lengths and sometimes as frequencies, and he continues to call the overtone series "harmonic", though fully aware that in terms of frequencies it is arithmetic [13]. Tartini argued, as we shall see, that difference-tones, his *terzo suono*, provided the link between his mathematics and the physical cause of musical pitch [14]. But his only defence against the objection that, by contemporary standards, his theory of proportion was upside-down [15], was an extraordinary argument based on the supposed areas covered by vibrating strings [16], an argument which shows that he had not even grasped that it is the frequency, not the amplitude, of vibration that determines pitch. Vallotti, who rejected any physical basis for musical theory, including the *terzo suono*, gave a more convincing reason for keeping to string-lengths, namely, that it was not possible to count vibrations, whereas it was possible to measure strings [17].

This obsolete identification of mathematically harmonic proportion with musically harmonic proportion resulted in Tartini's system linking directly with Zarlino's and, beyond him, with Greek musical theory. And, like any good Renaissance humanist, he claimed for his theories the authority of antiquity, together with the glory of being the first fully to restore this ancient, long lost, true science. Both his wonderful new mathematics, of which

[13] See e.g., Rameau, *Nouveau Système de Musique Théorique*, Paris, 1726, pp. 9-10, 14-5, 23 (Rameau, *Complete Theoretical Writings*, ed. E. R. Jacobi, n.p., 1967-72, Vol. II).

[14] V. infra p. 137.

[15] Serre, op. cit. (supra note 6), pp. 141-2.

[16] Tartini, *Trattato*, pp. 91-2; cf. Rubeli, ed. cit., pp. 140-1.

[17] Vallotti, *Della Scienza Teorica, e Pratica della Moderna Musica Libro Primo*, Padova, 1779, pp. 4-5, 58-9.

his musical theory formed only a small part, and his new theory of the diatonic genus, which was to explain all contemporary harmonic practice, had been known to Plato and, before him, to Pythagoras. But, like the Ancient Theologians, they had hidden the core of their knowledge and made public only its outer surface. Tartini had now rediscovered this precious core, developed it, and shown its conformity with modern science and modern musical practice. For he was aware that the ancient theory would require development, since Greek music was not polyphonic; the Greeks used only "successive harmony", not also "simultaneous harmony", as we do, though their diatonic scale, like ours, was based on consonances, namely, the harmonic and arithmetic division of the octave into C-f-g-c [18].

It seems likely that Tartini's interest in ancient music and the main source of his knowledge of it, apart from Zarlino and Vallotti, came from the Professor of astronomy at Padua, Gianrinaldo Carli, in whose house, in the 1740's, Tartini attended discussions on music among a circle of scientists and scholars [19]. Carli himself was a whole-hearted enthusiast for ancient music, which he believed to have been polyphonic and to have expressed and excited specific emotions, and he was contemptuous of modern music, which, with the exception of a few operas, was totally corrupt, aiming only at pleasure and amusement, and hence subject to fashion, like clothes. He believed that "music should be sentimental and not a meaningless, purely artificial arabesque"; and, according to him, these views induced Tartini to compose sonatas in a new style, such that like a "new Timotheus he excited within me, at his pleasure, various feelings, now of joy, now of sadness, now of fury" [20].

Tartini's views, however, on ancient music and its marvellous effects were those of a moderate musical humanist; which is what

[18] Tartini, *Diss.*, Prefazione (unpaginated); cf. *Risposta*, p. 8.

[19] Carli, *Delle Opere*, tomo XIV, Milano, 1786, p. 332.

[20] Ibid., p. 333: "la musica dee essere *sentimentale* e non arabesca, insignificante, e solamente artifiziosa"; "qual nuovo *Timoteo* eccitò a sua voglia dentro di me il sentimento vario or d'allegrezza, ora di tristezza, ora di furore". Cf. Capri, op. cit., pp. 195-6, 232.

one would expect from a reader of Zarlino. The effects must be accepted as historically true on the authority of such great men as Plato and Aristotle; for we have no existing monuments of the music of the ancients, and the details of their modal system are now unknowable. But the possibility of music producing striking emotional effects is proved by many of Tartini's personal experiences, of which he recounts one. In a *dramma* performed at Ancona in 1714, an unaccompanied bass recitative, of which the text expressed scorn (*sdegno*), produced such a violent emotional disturbance (*commozione di animo*) that the hearers changed colour and looked at each other in astonishment; and this effect was repeated at all thirteen performances. But such effects differ from ancient ones in two ways: first, they are the result of chance—there are no possible rules in modern music for producing them; second, they are only momentary, unlike the ancient effects, which could profoundly modify the hearers' character [21].

The basic reason why modern music cannot produce specific, lasting effects (the most it can do is to dispose the hearer to a genus of passions, not produce a specific emotion) [22] is that for us music has become an end in itself, dissociated from poetry and philosophy, whereas in antiquity musician, poet and philosopher were the same person [23].

> Music alone, separated from any other consideration whatever, has become our unique end and aim ("la musica sola, e disgiunta da qualunque altra considerazione si è fatta l'unico nostro fine, ed intento") [24].

and in fulfilling this aim of producing pleasing, satisfying music we have succeeded admirably. Nor does Tartini wish to revive the ancient effect-producing music [25]; nor does he think it even feasible—national differences, custom and tradition are such

[21] Tartini, *Trattato*, pp. 134-5.
[22] Ibid., pp. 141-3.
[23] Ibid., p. 145.
[24] Ibid., p. 150.
[25] Tartini, *Diss.*, p. 53.

important factors that probably the best ancient music would produce no effect on us [26].

Our use of polyphony is another cause of our failure to produce the effects [27]. Here Tartini revives the familiar sixteenth-century cancelling-out argument: high, quick sounds are joyful (*allegro*), and low, slow ones are sad (*mesto*); in polyphonic music these contrary effects cancel each other out, or, even if one claims that the soprano is the dominating part, the other voices will distract from its effect. It is strange that Tartini should apparently accept this crude theory, which ignores the possibility of chords having any specific character; for, when he discusses the various emotional qualities of melodic intervals, he very firmly makes the point that these qualities derive from harmony [28]. Everyone agrees that "the harmony of the major third is strong, joyful, bold; the harmony of the minor third is languid, melancholy and sweet", and this is because the harmonic division of the fifth, C - E♮ - G, is natural and strong, and the arithmetic division, C- E♭ - G, is less natural, and therefore weak. In the same way, a bass rising by a fourth or falling by a fifth is strong, and a bass rising by a fifth or falling by a fourth is weak, because of the harmonic and arithmetic division of the octave, C - G - C and C - F - C. And the parts above the bass will have the same character:

the rising semitone, E - F, is strong and joyful, and the descending one, F - E, is sweet and weak.

Like Vincenzo Galilei, but independently of him [29], Tartini considered that plainsong (*Canto Ecclesiastico*) had preserved something of the simplicity of ancient music, and might indeed be a descendant of Greek music [30], though he realized that the

[26] Tartini, *Trattato*, p. 151
[27] Ibid., pp. 141-3.
[28] Ibid., pp. 152-4.
[29] Tartini mentions Galileo Galilei (*Diss.*, p. 83), but only in a general context of modern science. If he read only Zarlino's *Istitutioni*, he would not be led to V. Galilei.
[30] Tartini, *Trattato*, pp. 144-5.

Church modes are not the same as the ancient ones [31]. Plainsong is monodic and its rhythm is free, without a regular beat, though, unlike ancient music, it does not observe the prosody of the text. The main difference between modern music and plainsong or sixteenth-century polyphony is the frequency and range of modulation; and this also is a contributory cause of our failure to produce the effects. His advice on modulation is very conservative: it should be mainly to the dominant and subdominant, with a preference for the former as being more natural (again because of the harmonic division of the octave); one should not modulate abruptly to a remote key, e.g. from a flat to a sharp key; one should keep in the tonic more than in any other key. Tartini is doubtful whether all this modern modulation is natural. Popular song modulates very little, and he has noticed that, when a long pedal-note on the dominant or tonic occurs, usually towards the end of a piece, the audience's attention is aroused—this is because the tonic is thereby established [32]. It is odd that a practising musician and composer should in this context make no mention of the expressive possibilities of modulation, which another independent thinker, J. A. Ban, had adumbrated over a century before [33].

Another basic respect in which Tartini is consciously archaic is his use of a threefold scheme of knowledge, of *scienza*, a scheme that becomes explicit in the *Dissertazione*, and indeed forms the framework of the whole treatise. The three genera of knowledge are, in Tartini's order [34]:

1) physical, knowledge of the given facts of nature; what we would now call empirically established scientific knowledge; this is practised by the learned moderns (*i Dotti moderni*).

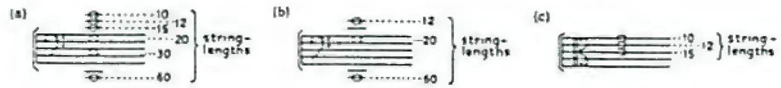
2) demonstrative, knowledge obtained by operation of the intellect, that is, mathematical truths, which Tartini assumes to be absolutely certain; this was practised by the ancient Greeks.

[31] Ibid., pp. 138-140.
[32] Ibid., pp. 146-8.
[33] Cf. supra pp. 90-91.
[34] *Diss.*, p. 17; cf. *Trattato*, pp. 20-1.

3) musical, knowledge based on sense-experience (*il senso*) and general consent (*consenso commune*), that is, facts and judgments concerning the practice of music accepted by all experienced musicians; this is practised by the professors of the art of music.

All certainly true propositions about music will be verifiable in all three genera. Any proposition that does not satisfy this condition will be either false or "of opinion only"; Tartini evidently has in mind the Platonic distinction between ἐπιστημη and δοξα. The three genera are a hierarchy, in descending order: demonstrative, physical, musical, in respect of certainty, generality, and, I think, causal priority. Since in Tartini's own certainly true system all three should always be present all together, this hierarchization, which is not explicit, is complicated and can best be seen from an example, a basic one which runs through the whole of the *Dissertazione* [35].

There are three aspects of the major chord:



a) is based mathematically on the harmonic series $1, \frac{1}{2}, \frac{1}{3} \dots \frac{1}{6}$, i.e. Zarlino's *senario* or *sestupla*; physically on the *terzo suono* (not, as one would expect, on overtones; why this is so will appear later); musically it gives all the consonances actually used [36]. It is therefore the most general form of the chord, from which b) and c) derive.

b) has primarily a physical basis, namely, the overtones produced by a string, which Tartini believed, wrongly, were only the third and fifth partials (though in the *Diss.* he is willing to concede the presence of even partials, and calls the third and fifth partials *suoni dominanti*). In string-lengths these are $1, \frac{1}{3}, \frac{1}{5}$.

c) is founded on musical practice, being the major triad, which is assumed where a *basso continuo* is unfigured. If it is figured, it is thus: $\frac{5}{3}$.

[35] Ibid., p. 28.
[36] Except the minor sixth; on which v. infra p. 155.

b) and c) derive from a) for the obvious reason that their notes are present in a); but also they derive mathematically. In terms of string length c) is $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, which is the last part of the a) series 1, $\frac{1}{2}$, $\frac{1}{3}$ . . . $\frac{1}{6}$. b) is 1, $\frac{1}{3}$, $\frac{1}{5}$, itself a harmonic proportion, which also derives from the a) series in that $\frac{1}{3}$ is the harmonic mean between $\frac{1}{2}$ and $\frac{1}{4}$, and $\frac{1}{5}$ between $\frac{1}{4}$ and $\frac{1}{6}$. Up to this point, the three genera of knowledge still seem to be on a level: a) is at the top of the hierarchy only because it embraces all three kinds. And, apart from errors concerning difference-tones and partials, the argument is fairly clear and sane. But in what follows the primacy for Tartini of demonstrative or mathematical knowledge becomes evident, his reasoning becomes obscure and arbitrary, and his style elliptical and passionate. This is a pattern we shall see again and again.

Tartini considers that the above mathematical derivation of b) and c) from a) shows that b) is a mean between the two extremes a) and c); but he also adduces another proof of a kind, he claims, quite unknown hitherto to the mathematics of proportion [37]. If you take the reciprocals of b), you have the arithmetic proportion 1, 3, 5; this has always been known; "but it is not known, and has never been known, that, as this proposition is true of three terms, so it may also be of three given proportions". He then adds up the string-lengths of a), b) and c), which give respectively the sums: 147, 92, 37. But these are in arithmetic proportion, "Adunque ec.". As so often, he here ends his demonstration with the Euclidean Q.E.D., but without having enunciated his theorem, so that the exasperated reader is left still wondering *quid esset demonstrandum*. In this case, however, it is fairly clear that the point is to show that b) is some kind of mean between a) and c), and this in turn is supposed to prove that the physical bases of a) (*terzo suono*) and b) (string overtones) are of the same nature—but how or why we are not told. The reasoning is arbitrary in the extreme. Starting from the true statement that the reciprocals of a harmonic proportion give an arithmetic one, Tartini then adds together *not* the reciprocals of a), b) and c)

[37] *Diss.*, p. 29.

but the original string-length fractions (expressed in whole numbers). If he had added together the reciprocals, b) would have become an extreme and c) the mean : a) $(1 + 2 + 3 + 4 + 5 + 6)$ 21, b) $(1 + 3 + 5)$ 9, c) $(4 + 5 + 6)$ 15. Nor does he attempt to explain why a significant result should be produced by adding the terms of ratios and then comparing them.

Tartini saw himself as fighting a battle against two classes of opponent, which correspond to the genera of knowledge 1) and 3) [38]. First, those scientists who seek to explain musical sound in purely physical, mechanistic terms, based on empirical knowledge of the vibrations in the air and in sounding bodies. Second, those theorists who claim that music is a matter only of feeling and the instinctive judgment of the ear. Both these classes neglect genus 2) of knowledge, the demonstrative-mathematical, regarding it as superfluous and irrelevant. Tartini, therefore, though his main aim is to combine all three kinds of knowledge, is throughout his works particularly concerned to defend the validity of the purely mathematical genus and to assert the absolute necessity of including it in any complete musical theory. It might then be thought that his emphasis on the importance of mathematical explanation is due only to this defensive attitude. But this is not, I think, the case, and that he does attribute a real priority to the mathematical genus of knowledge over the other two will become clear as we examine his theories in detail, especially those concerning the minor mode and temperament. He regarded the relation of physical-mechanistic explanations to mathematical ones as being the same as the relation of the Aristotelian material cause to the formal cause, and the latter is "as positive and real" as the former, "if not more so" [39].

Occasionally Tartini introduces a fourth kind of knowledge (*scienza*), namely, metaphysical, based, I think, on the presupposition that the universe is wholly ordered and intelligible [40]; this

[38] Ibid., pp. 112-3.
[39] Ibid., p. 113.
[40] Ibid., p. 64; cf. *Trattato*, p. 32.

is plainly prior to the other three. For example, when justifying his use, to explain the minor mode, of the series of residues of a string divided harmonically, i.e. (1,) $\frac{1}{2}$, $\frac{2}{3}$ . . . . . $\frac{5}{6}$, derived from 1, $\frac{1}{2}$, $\frac{1}{3}$ . . . . $\frac{1}{6}$, he claims that "all the genera of knowledge" show that the former series is as systematic as the latter. The metaphysical argument is: if one series of parts of a homogeneous whole (the string) gives a significant, harmonic result (the harmonic series), the series of remaining parts (residue series) cannot possibly be "useless and idle" (*inutile, ed oziosa*). He then runs through the arguments drawn from physical, mathematical, and musical knowledge [11].

It is clear that the third kind of knowledge, that based on musical practice, is of a lower epistemological status than the other two (or three, if we include the metaphysical genus) in respect of both certainty and generality. It is based on a supposed conformity of human behaviour at all times and in all places—the same kind of argument that was used in natural theology as one of the proofs of God's existence: the *consensus gentium*. This conformity shows that the behaviour in question is natural, in the sense of being a universal psychological datum that can be altered only violently and temporarily. A supremely important example for Tartini of this kind of natural datum (*verità di natura*) is the diatonic genus, which has remained the same from pre-Platonic times until his own day [12]. But Tartini is, as we have seen, aware of major historical developments in music, so that the supposed conformity of human behaviour must be limited to allow for such changes. The required limitation is supplied by his demand that the three kinds of knowledge must converge on to a single truth: only those elements of musical practice are natural and remain constant which are based on mathematical and physical truths. Greek music was monodic, part of poetry and philosophy, and effect-producing; ours is polyphonic, an end in itself, and purely pleasurable. The natural element that has remained constant is the diatonic scale, mathematically derived from the harmonic and arithmetic divisions of the octave, and now physically confirmed by the *terzo suono* and overtones.

[11] Cf. infra p. 156.
[12] *Diss.*, Pref.

This third kind of knowledge can also be used in a non-historical way to confirm truths of the other two kinds [13]. For example, Tartini states that he accepts the following propositions from the article on fundamental bass in the *Encyclopédie* [13a]:

(i) Untaught people can sing the diatonic scale correctly.
(ii) Untaught people with a good ear sometimes improvise a satisfactory bass to a given tune.
(iii) That this "easy, natural" singing of the scale is "suggested by the fundamental bass".

(i) and (ii), claims Tartini, are confirmed by the experience "of the whole human species", and (iii) is a legitimate deduction from them. Thus Tartini's mathematical and physical explanations of the diatonic scale and its fundamental bass are corroborated by the natural, untaught behaviour of the whole human race. The difference between what is natural and constant, on the one hand, and what is due to art and may evolve historically, on the other, is seen in two possible basses to the upper parts

The fundamental bass is

(perfect cadence)

while musical art also suggests

(interrupted cadence)

The untaught, natural singer will always improvise the former and never the latter.

[13] Ibid., pp. 110-1.
[13a] *Encyclopédie*, ed. Diderot & d'Alembert, t. 7, Paris, 1757, p. 60 (art. Fondamental).

ii. The Physical Basis

Since Tartini did in a large measure succeed in his aim of unifying the three kinds of knowledge, it is perhaps misleading, in expounding his system, to take them one by one; but in the interests of clarity it is necessary, and I shall here follow Tartini's own order in both his treatises: first the physical basis, then the mathematical basis, and finally musical practice.

Chapter I of the *Trattato*, entitled "De' Fenomeni Armonici", contains several curious errors, some of which had an important effect on his whole musical system, and some of which he corrected in the *Diss.*, after they had been pointed out by Jean de Serre. With regard to overtones, as has already been mentioned, Tartini believed, at the time of the *Trattato*, that a string produced only the third and fifth partials. If he was experimenting by himself, this would be an easy mistake to make, since the even partials, being octaves of lower notes, are much more difficult to hear [44]. It is nevertheless odd that he was not led to correct it either by experiments with sympathetic vibration, such as Descartes had made long ago, or by his knowledge of how such instruments as the tromba marina or the trumpet are played, namely, by making them sound a complete series of overtones. It is still odder that a professional violinist and teacher of the violin, writing nearly twenty years after the publication of Mondonville's *Les Sons Harmoniques* [45], should be ignorant of harmonics on the violin [46]. By the time of the *Diss.* Tartini knows, from Rameau and the *Encyclopédie*, that it is generally, and correctly, believed that strings and many other sounding bodies produce even as well as odd partials; but he is still not convinced, and retains as a separate category the string producing only the third

[44] Rameau (*Nouv. Syst. de Mus. Th.*, 1726, pp. 17-8) admits that even partials are extremely difficult to hear, but considers their existence proved by sympathetic vibration and by the use of organ stops sounding the complete series of overtones.

[45] *Sonates à violon seul*, Paris, circa 1735.

[46] That he did not know them and had not tried to produce them is certain from the erroneous statement (*Trattato*, p. 10), based on a mistaken theory of vibration, that, if on a tromba marina, the finger is lightly placed $\frac{2}{3}$ of the string away from the nut, no musical sound is produced. Cf. Serre, *Obs.*, pp. 113-6.

and fifth partials, though he now makes the concession of calling them *suoni dominanti*. The consequence of this obstinacy is that he does not consider overtones a sufficient physical basis for the harmonic series 1, $\frac{1}{2}$, $\frac{1}{3}$ . . . . . $\frac{1}{6}$, which he had taken over from Zarlino, and instead, bases the series on difference-tones, his *terzi suoni*.

This phenomenon is easy to observe in double-stopping on a violin. If you play a third or a sixth fairly high up and loudly, you will hear a lower note of a rather bumbling quality but of a definite pitch; the frequency of the lower note will be the difference between the frequencies of the two higher notes, e.g. c'' - e'' two octaves above middle C will give middle C as their difference-tone; g' - e'' will give the c' above middle C; e'' - c''' will give the g' a twelfth above middle C (4:5 gives 1; 3:5 gives 2; 5:8 gives 3). We may then believe Tartini when, in the *Diss.*[47], he claims to have discovered difference-tones by chance early in his life, in 1714, when at Ancona. He communicated his discovery to other musicians and later used it as a guide to intonation when teaching at the violin-school he started at Padua in 1728. He uses it extensively in the *Trattato*, but without claiming to have discovered it. He later made the claim because Serre had noted that some French musicians had independently discovered difference-tones and published their discovery just before Tartini's *Trattato* appeared [48]. Serre also pointed out that Tartini's difference-tones were all an octave too high, an error which he corrected in the *Diss.*[49]. But Serre failed to notice another mistake, presumably because he himself also made it, namely, that Tartini gives the wrong *terzo suono* for the major and minor sixths: for g' - e'', middle C instead of c' (but rightly c' in the *Trattato*, where all the others are an octave too high); for e'' - c''', middle C instead of the g' a twelfth above [50]. Tartini's formula, first given in the *Diss.*, for finding the string-length of the difference-

[47] *Diss.*, p. 36.

[48] Serre, *Obs.*, pp. 86-7.

[49] Ibid., pp. 83-4, 119.

[50] Tartini, *Trattato*, pp. 13-8; Serre, *Obs.*, pp. 84-5.

tone resulting from any two-note chord is: multiply together the two terms of the ratio of the chord and the product will be the *terzo suono*. This formula gives correct results for all the consonances having superparticular ratios (e.g. a major third 4:5 gives 20; 20:5:4 = 1:$\frac{1}{4}$:$\frac{1}{5}$), so that in the natural harmonic series

1 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ ($\frac{1}{8}$)

any two successive notes sounded together will give the fundamental as their difference-tone, whereas Tartini wrongly believed that any two notes whatever of the series would produce the fundamental (e.g. a tenth c' - e'' (2:5), which in fact gives g' between the two notes). Presumably he worked experimentally through the consonances in their usual order as far as the minor third, and, having found that these all gave the fundamental as *terzo suono*, made up his formula and did not bother to test the sixths or tenths.

But for Tartini's purposes the mistake was not an important one. The phenomenon of difference-tones does in fact reinforce the overtone series; and it is perhaps legitimate to argue, as Tartini did [51], that it provides a surer physical basis for the ratios of consonances than does the overtone series. For not all musical instruments produce a complete series of overtones, and the relative strength of them varies of course greatly from one instrument to another, whereas difference-tones, being, as we now think, caused within the ear, are independent of the source of sound and will always be the same whatever the nature of that source. Tartini did not think that the *terzo suono* was subjective; he never goes into any details about its causation, but states vaguely that it results from the collision of two vibrating bodies of air, and this allows him to argue that difference-tones remain the same whatever the nature of the sounding bodies producing the two generating sounds [52].

[51] Tartini, *Diss.*, p. 112.
[52] *Trattato*, pp. 13, 56.

There is another group of errors in the first chapter of the *Trattato*, also noticed by Serre [53], which need not detain us long, since it has no consequences in the rest of his system. Among the "fenomeni armonici comunemente noti" Tartini lists the oscillations of vibrations of "corde pendole", without making it clear whether he is referring to pendulums or vibrating strings. He gives the following two laws, which derive ultimately, the first from Galileo Galilei and the second from Vincenzo Galilei [54]:

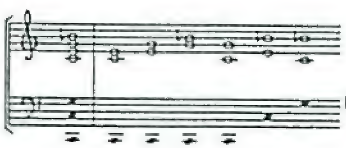
(i) If *corde pendole* have equal weights attached to them and their lengths are in the ratios 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$ . . . , they will produce the sounds of the harmonic series 1, $\frac{1}{2}$, $\frac{1}{3}$, ... (string-lengths).
(ii) If *corde pendole* of equal length have weights attached to them in the ratios 1, 4, 9, 16 . . , they also will produce the sounds of the harmonic series.

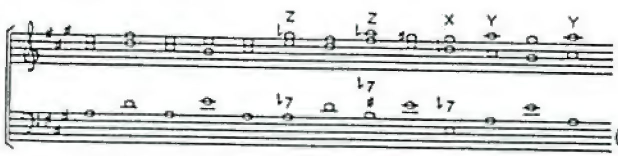
(i) is true for the oscillations of pendulums, though the weights are irrelevant and pendulums do not produce sounds; it is false for vibrating strings, for which the lengths would be those of the harmonic series. (ii) is true for vibrating strings and false for pendulums.

All these errors suggest that Tartini's knowledge of musical acoustics, at least when writing the *Trattato*, was acquired by word of mouth, probably from Vallotti and from Carli and his circle, occasionally supplemented by experiments, which were unsystematic and sometimes misleading. One field in which he did make musical experiments of great originality and interest was the use of the natural seventh, i.e. the seventh partial, which from a fundamental C gives a very flat B♭. The ratio of this natural seventh is 4:7, which is narrower than E-d (fifth + minor third), 5:9, by 35:36, and narrower than G-f in the Ptolemaic just scale (fifth + minor tone + semitone), 9:16, by 63:64, while G-f is narrower than E-d by a comma, 80:81. Ptolemy had given scales in which ratios involving 7 are used, and Mersenne had

[53] Tartini, *Trattato*, pp. 12-3; Serre, *Obs.*, pp. 117-8.
[54] V. supra pp. 31, 23-4.

suggested that through long custom such ratios might come to be accepted as consonant [55]. But no one, as far as I know, before Tartini had categorically stated that the natural seventh is consonant [56], or pointed out the obvious connexion between it and the privileged position of the dominant seventh compared with other dissonances, namely, that it need not be prepared and does not resolve on the same bass. In accordance with his erroneous theory, Tartini believed that the *terzo suono* of the natural seventh c'' - b♭'' was C (in the *Diss.*; c' in the *Trattato*), whereas in fact it is g' (4:7 gives 3, not 1 or 2). But this mistake is of little importance, since he is considering the use of the whole chord of the seventh, and the true difference-tones of this would give a strong fundamental C [the sign ♭ is used by Tartini for the natural seventh]:



Just after his first exposition of the *terzo suono* in the *Trattato* [57] Tartini asks the question: "What is the relation of the *terzo suono* to the intervals from which it results?", and he answers it by the following example:

(The difference-tones are an octave too high; the chord at X would give a B, and the chords at Y an E, but the full chords would give Tartini's bass).

[55] Mersenne, *Harm. Univ.*, I des Cons., pp. 87, 89.
[56] Rameau rejected it; see Matthew Shirlaw, *The Theory of Harmony*, London, n.d., pp. 78, 163.
[57] Tartini, *Trattato*, pp. 17-8.

Of this he states [58]:

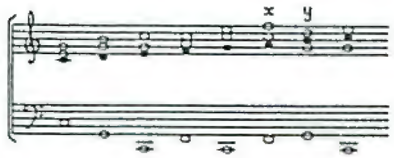
> Given these intervals, under which is placed their respective *terzo suono*, this is demonstratively the harmonic Bass of the given intervals, and any other bass whatever will be a paralogism.

He then explains that the thirds at Z are of the ratio 6:7, which differ from a minor third (5:6) by 35:36, and that D♭ at X is the natural seventh of the bass E. He then continues [59]:

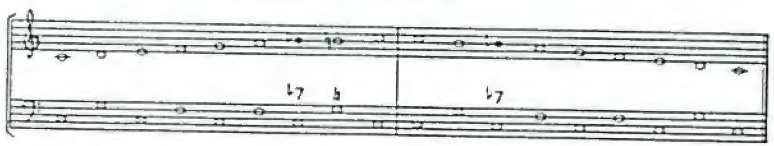
> This interval is extremely easy to play in tune on the violin; it is intended by harmonic nature, because it is made by nature on the tromba marina, the trumpet and the hunting-horn.

By "Harmonic Bass" Tartini means what Rameau and d'Alembert called the fundamental bass, and he himself uses the latter term later in the *Trattato* and throughout the *Diss.* The term, strictly speaking and as defined in d'Alembert's article in the *Encyclopédie* [60], means the bass notes which, as fundamentals, generate chords made by the upper partials, so that, e.g., the fundamental bass of the chords G-c-e or E-g-c or g-e is a low C; and, by the French theorists, the fundamental bass was applied to minor chords, and even chords of the seventh and added sixth. The fundamental bass, therefore, to any upper part or parts produces chords in root position. With a few exceptions, it does not move by step, but by consonant intervals, mainly the fourth and fifth. Tartini, except for the above example and a few others to be discussed shortly, confines himself to giving the fundamental bass of the major diatonic scale; and for him, of course, the fundamental bass derives, not from the overtone series, but from the *terzo suono* (it also has a mathematical derivation, which I will deal with later). The scale is harmonized thus [61]:

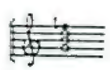
[58] Ibid., p. 17: "dati i seguenti intervalli, de' quali è rispettivo terzo suono il sottoposto, questo sarà dimonstrativamente il Basso armonico de' dati intervalli, e sarà paralogismo qualunque altro Basso vi si sottoponga".
[59] Ibid., p. 18: "Questo intervallo è di facilissima intonazione sopra il Violino; è voluto dalla natura armonica, perchè si trova fatto dalla natura nella tromba marina, trombe da fiato, e corni di caccia".
[60] *Encyclopédie*, ed. Diderot & d'Alembert, t.2, Paris, 1751, p. 119 (art. Basse, sect. Basse Fondamentale).
[61] Tartini, *Diss.*, p. 80.



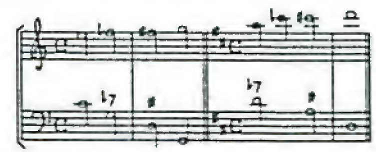
The weak spot in this harmonization, as Tartini himself realizes, is the subdominant-dominant progression at x y, where he has had inconsistently to put the G below the B in order to avoid consecutive fifths and octaves. According to Tartini, this fault is still worse in the descending scale, where the tritone B♮ - F at y x is more offensive because the progression is unnaturally moving from the more perfect, dominant chord to the less perfect subdominant. Both these defects could be remedied by inserting the natural seventh into the scale, thus [62]:



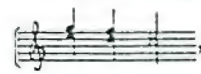
The bass is now a palindrome, and the descending scale is as good as the ascending. If one took the natural seventh chord as a dominant seventh, the descending scale would be satisfactory and the ascending one very odd. But Tartini does not so take it. He is most emphatic that the natural seventh chord is consonant:

> given these notes , I say that, combined in any way whatever, they will always have G as their *terzo suono*, which is their harmonic fundamental Bass. Therefore such a seventh is consonant, not dissonant. Therefore it need not be prepared, nor resolved; it can rise or fall, and, if the intonation is just, it will sound equally well [63].

He gives two examples of it rising [64]:

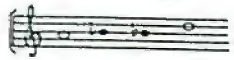
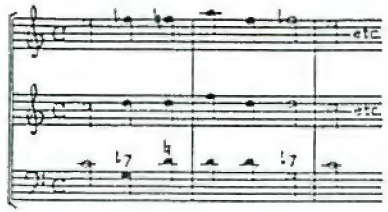
[62] *Trattato*, pp. 131-2; *Diss.*, pp. 91-2.
[63] *Trattato*, pp. 128-9: "date in armonia queste note . . . . , dico, che comparate in qualunque modo tra loro, si avrà sempre terzo suono Gsolreut, ch'è il Basso armonico fondamentale. Dunque una tal settima è consonante, non dissonante. Dunque non ha bisogno di esser apparecchiata, nè di esser risoluta; può ascendere, e può discendere; e quando la intonazione sia giusta, starà egualmente bene".
[64] Ibid., p. 129.

Tartini does, however, connect the natural seventh chord with the dominant seventh. This connexion is of course assumed in the first example above. After assuring us that we should have "no scruple" in using the above progression "when necessary", he continues [65]:

> The best of it is that more or less the same is done in common practice. On the dominant, which forms the harmonic [perfect] cadence with the tonic, the seventh is used without being prepared. In final cadences it is the almost universal practice to add the seventh to the penultimate note. There is no rule here, rather it is against the rule, since such a seventh is not prepared. But nature is stronger than art.

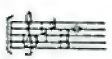
The difference between F in C major and the natural seventh above G is so slight (63:64) that the ear is little, or not at all disturbed when F is added to the triad G-b-d. "Hence, in spite of any rule, there is not and cannot be, for the upper parts , any other harmonic Bass than  "[66].

One can see here, though Tartini does not mention it, a good reason for insisting on the use of the dominant seventh in such a case: the bass provided by the *terzo suono* (or by the overtone series) would be the impossible . This fact makes it all the more extraordinary that Tartini, when giving rules for the use of dissonances, does not, as we shall see, give a privileged place to the dominant seventh [67].

[65] Ibid.: "Il meglio si è, che appresso a poco si fa lo stesso dalla pratica comune Alla quinta nota del tuono, che formi cadenza armonica con la nota principale del tuono, si dà la settima senz'apparecchiarla. Nelle cadenze finali è uso quasi generale dell'accompagnamento organico di aggiunger la settima alla nota, che propone la cadenza finale. Regola non vi è, anzi è contro la regola, perchè tal settima non è apparecchiata. Ma la natura ha più forza dell'arte".
[66] Ibid.: "Indi ne viene, che ad onta di qualunque regola date le due parti . . . . . , non vi è, nè vi può esser altro Basso armonico se non questo . . . .". The other good Bass, C D A, is in the minor mode.
[67] But cf. infra p. 99.

In the *Trattato* this defence of the natural seventh is introduced in the context of the ancient Greek enharmonic genus. Tartini suggests the enharmonic tetrachord , where the first B is the natural seventh above C, which can be satisfactorily harmonized thus (transposed up a fifth "per l'effetto più sensibile"):

When this was played on two violins and *basso continuo*, "in my experience, and by the common consent of the Professors admitted to the experiment, the effect was so pleasing that nothing could be better" [68].

Another instance of Tartini's actually trying out the use of the natural seventh is a short four-part piece composed in a new scale, ultimately derived from the series $1, \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}$:

in which he uses only four chords: the triads A minor, F major, E major and the chord . He argues that this last chord is also consonant, because F♮ - D♯ is so near to the natural seventh, the difference being only 224:225. In the piece he treats this chord, not as a dominant seventh, but as an augmented sixth, that is to say, it always resolves outwards on to E major. The piece produced an excellent effect (*ottimo effetto*) both on his "and other dispassionate ears" [69].

In spite of his great interest in the natural seventh, his experiments in its practical use, with their favourable results, his realization of its connexion with the dominant seventh, and his awareness of its usefulness for his fundamental bass, Tartini is anxious to exclude it from the diatonic genus and to preserve Zarlino's harmonic series $1, \frac{1}{2}, \frac{1}{3} \ldots \frac{1}{6}$, which stops short before $\frac{1}{7}$—indeed, as we shall see, he goes to enormous lengths to provide mathematical reasons why the series should stop here. Also, it is not clear how seriously he was advocating a revival of the enharmonic genus, which, in his version of it, would entail the use of the natural seventh, or the practical use of the new scale, just described above. In the later treatise, the *Diss.*, he seems on the whole much less favourable to any ratios involving seven; but this may well be only because the *Diss.* is a treatise about the diatonic genus.

[68] *Trattato*, pp. 127-8: "per mia sperienza, e per consenso comune de' Professori ammessi alla sperienza . . . . l'effetto è talmente grato, che nulla più".
[69] Ibid., pp. 260-3.

iii. The Mathematical Basis

I. The Circle

In the *Trattato* the circle plays a predominant and essential rôle as the mathematical foundation of the whole of Tartini's system, a system which embraces fields far wider than musical theory, though of these we are given only hints in his printed works. In the *Diss.*, the circle has almost completely disappeared; but this is not because Tartini has abandoned it, but because he deals with it in another work published simultaneously, a *Risposta* to his critic, Jean de Serre [70].

Of the very long second chapter of the *Trattato*, entitled "Del Circolo, sua natura, e significazione", Jean de Serre wrote: "Je ne sai s'il s'est trouvé aucun Lecteur qui ait pu soutenir la lecture entière de ce chapitre" [71]. I can claim to be such a reader, but certainly not to have understood it all. That this part of Tartini's system is extremely obscure, he was himself aware; and both at the end of the *Trattato* and in the *Risposta* he excuses this difficulty and obscurity as being inevitable in the exposition of such completely novel and profound theories [72]. But from his private

[70] V. supra note 6.
[71] Serre, *Obs.*, p. 121.
[72] Tartini, *Trattato*, p. 171; *Risposta*, pp. 7, 12.

correspondence with Padre Martini about the *Trattato*, of which he had sent the manuscript in 1751, it is clear that Tartini was being deliberately obscure, wishing to convey an esoteric message to only a chosen few. I will return later to this important point [73]. One reason why his exposition is both so prolix and so difficult to follow is that he does not know algebra, as he himself says [74], so that all his complicated juggling with ratios and proportions is carried out in numbers, which are constantly being multiplied to avoid fractions and then divided to show the ratios in their lowest terms. As much as I have been able to grasp of his circle theories I will give in algebraic terms.

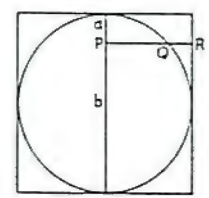
The main purpose of the whole chapter is to demonstrate that the circle is "intrinsically harmonic", harmonic in the mathematical sense, so that, as already pointed out, the connexion with musical ratios disappears in an age thinking in frequencies and not string-lengths. But, even if we overlook this basic archaism, Tartini's demonstrations seem arbitrary and unconvincing. The circle is chosen because, of all geometric figures, it is the most monadic (*uno in se stesso*), and this is because all its radii are equal. The circle must be inscribed in a square, because straight lines are prior to curved ones; a circle cannot be constructed without a radius [75].

Before attempting to understand Tartini's demonstration of the harmonic nature of the circle, we must first look at the *Trattato Premesso*, with which the book begins and which, he says, is a "piccola parte della scienza aritmetica antica". This is concerned with what he calls a discrete geometric proportion (*proporzione geometrica discreta*), i.e. a proportion of the form $a:b = c:d$, as opposed to a continuous geometric proportion of the form $a:b = b:c$. He introduces it as a method of approximating in rational numbers to an irrational geometric mean. The purpose of doing this is not, as one might expect, for practical use in temperament,

[73] V. infra p. 169.
[74] *Trattato*, p. 27.
[75] Ibid., p. 21.

e.g. as a means of dividing a tone or a major third in half, but for use in his circle theory, and, in the *Diss.*, as we shall see, for his fundamental bass. The simplest form of the procedure is as follows. Between two extremes $a < b$ take the arithmetic mean $\frac{a+b}{2}$; multiply the terms a, b, $\frac{a+b}{2}$ together so as to produce the four terms: $a\frac{(a+b)}{2}$, ab, $\left(\frac{a+b}{2}\right)^2$, $b\frac{(a+b)}{2}$. The geometric mean, $\sqrt{ab}\,\frac{(a+b)}{2}$, lies between the two middle terms, which are respectively the harmonic and arithmetic means of the new extremes. The four terms can be more simply expressed: a, $\frac{2ab}{a+b}$, $\frac{a+b}{2}$, b (geometric mean: $\sqrt{ab}$, which is also the geometric mean of the harmonic and arithmetic means); but this would not suit Tartini's manipulation of the circle.

Tartini considers that the sines of the circle exhibit its specific nature, whereas it has its diameter and radius in common with the exscribed square. The sines are shown to be intrinsically harmonic by the following procedure [76].



Divide the diameter unequally into $a < b$ at P. Draw the sine from P to Q, and continue it to meet the side of the square at R, so that PR equals the radius. Then (by Euclid, VI, 13) $PQ = \sqrt{ab}$, and $PR = \frac{a+b}{2}$. Now form the discrete geometric proportion from a, b, and you have the above four terms, $a\frac{(a+b)}{2}$, ab,

[76] Ibid., p. 22.

$\left(\frac{a+b}{2}\right)^2$, $b\frac{(a+b)}{2}$. The harmonic mean, ab, is the sine squared; the arithmetic mean, $\left(\frac{a+b}{2}\right)^2$ is the radius squared (and the geometric mean, $\sqrt{ab}\,\frac{(a+b)}{2}$, is the sine multiplied by the radius; which Tartini does not here mention). Tartini has thus succeeded in making the sine into a harmonic mean, but only by squaring it and multiplying the other terms by the radius, and he has shown that the radius, which belongs also to the square, is always the arithmetic mean. What is in fact given in the figure is that the sine is the geometric mean of the two extremes of the divided diameter; one could therefore argue that the circle is intrinsically geometric. Tartini himself raises this objection [77] and, if I have understood him correctly, answers it thus [78].

Since it is true that harmonic mean times arithmetic mean = (geometric mean)^2 ($\frac{2ab}{a+b} \times \frac{a+b}{2} = ab$), or $H = \frac{G^2}{A}$, and since G = sine and A = radius, which, combined, generate the circle, one can say that the circle is intrinsically harmonic because the sine and radius combined $\left(\frac{G^2}{A}\right)$ produce the harmonic mean (H). The sine and radius generate the circle in that one can conceive of a quadrant of a circle being built up of an infinite number of sines deriving from an infinite number of divisions of the diameter, the length of the sine being determined by the circumference traced by the revolving radius. Moreover, the sine, being the geometric mean both between the extremes of the unequally divided diameter and between the harmonic and arithmetic means of the same extremes ($\sqrt{ab}$ is the geometric mean both of a, b, and of $\frac{2ab}{a+b}$, $\frac{a+b}{2}$), partakes of the nature of both the harmonic and the arithmetic means. "This proposition", says Tartini, "is in itself, and independently of the musical system, of such and so great importance, that I am sure there is not its equal in all known human sciences" [79].

[77] Ibid., pp. 36-7.

[78] Ibid., pp. 38-9.

So far the reader, though perhaps bewildered and unconvinced, may have some understanding of Tartini's argument. But in what follows I cannot hope even for this modest result. Still with the aim of showing the circle to be intrinsically harmonic, Tartini now introduces the ratio of the circumference to the diameter, i.e. π. He starts from the objection [80]: if the circle is intrinsically harmonic, the radius, diameter and circumference should be a harmonic triad, but 1, 2, 2π are not. Moreover it is demonstrable that there is no harmonic series beginning 1, 2, because, if 1, 2, x were such a series, then x:1 = x-2:1, which is absurd. But it is true that, as x approaches infinity, so 1, 2, x approaches being a harmonic series. Tartini then applies to x the term "indefinito", using it to mean, first, "infinite", as in the previous sentence, and then "irrational" as in the following statement [81]:

> the proposition is true, that 1, 2, x must fit the circular figure in virtue of its nature and construction; and that the indefinite term x is the circumference, which with the term 2 forms an indefinite ratio. This ratio is in substance a transcendental harmonic ratio, composed of the ratio 1:3, as being the fundamental ratio of the harmonic system, and of the two respective centres of the two ratios forming 1:3, namely, 1:2 and 2:3.

I shall now attempt to explain this statement.

[79] Ibid., p. 39: "Oltre che questa proposizione è il fondamento del mio sistema, si degni di credermi, che la proposizione è per se, e indipendentemente dal sistema musicale di tal, e tanta importanza, che son sicuro non esservi la eguale in tutte le note umane scienze".

[80] Ibid., pp. 27, 32.

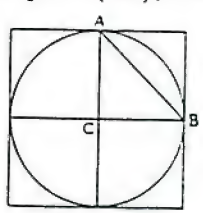
[81] Ibid., p. 40: "Dunque vera la proposizione, che 1, 2, x deve adattarsi alla figura circolare in forza della sua natura, e construzione; e che il termine indefinito x sia la circonferenza, quale col termine 2 forma una ragione indefinita. Questa ragione in sostanza è una ragione armonica trascendente, composta dalla tripla, come ragione fondamentale del sistema armonico, a da due centri rispettivi delle due ragioni formanti la tripla, cioè dupla, e sesquialtera." On this cf. Rubeli, ed. cit., pp. 101, 112. Tartini returned to this matter in his *Risposta* (pp. 37-42); his arguments are extremely obscure, and, I think, deliberately evasive.

The ratio 1:3 is the fundamental ratio of the harmonic system because it determines the harmonic series [82]; that is to say, given a series beginning 1, $\frac{1}{2}$, it is not until we put $\frac{1}{3}$ that we know that the rest of the series will be harmonic, as opposed to 1, $\frac{1}{2}$, $\frac{1}{4}$ .... or 1, $\frac{1}{2}$, $\frac{2}{3}$ .... In musical terms 1:3 is a composite interval, a twelfth, made up of an octave, 1:2, and a fifth, 2:3. The contention is that π, which is approximately $\frac{22}{7}$, is made up of $\frac{21}{7} = \frac{3}{1}$ plus an irrational quantity, somewhat less than unity, which is composed of the "centres" of 1:2 and 2:3. By the "centres" of these ratios Tartini means some kind of irrational root of them, to which he approximates by his discrete geometric proportion, in this case elaborated to include the contraharmonic mean as well as the harmonic, arithmetic and geometric means.

Tartini has already, before giving this answer, tried to justify his use of roots and powers of ratios, instead of simple ratios [83]—one of the most strikingly unsatisfactory features of his circle-system. This justification consists of the proposition [84]:

> The first harmonic principle does not fall under the laws of harmonic differences [i.e. ordinary harmonic proportions, which are based on the differences between the terms], but it falls under the laws of the duple power (*potenza dupla* [i.e. $\sqrt{2}$]), which is *a priori*.

This proposition is proved by two demonstrations. First [85], that a quarter of an inscribed circle and a quarter of an exscribed square have in common the radius of the circle and the side of the square (CB), and also the chord in the quadrant of the circle and the diameter of the square (AB); these are in the ratio $1:\sqrt{2}$.



[82] *Trattato*, pp. 30-1.
[83] Ibid., pp. 34-6.
[84] Ibid., pp. 35-6: "il principio primo armonico non cade sotto le leggi delle differenze armoniche, ma cade sotto le leggi della potenza dupla, ch'è *a priori*".
[85] Ibid., p. 22.

Second [86], that the ratio, circumference: circumference minus radius, approximates to $2^{1/4}$, i.e. the square root of $\sqrt{2}$. Tartini proceeds by taking the differences between the radius, diameter and circumference. In terms of the approximation for π $\frac{22}{7}$, the latter are 7, 14, 44; the differences, 7 and 30, are then added together and compared with the circumference, giving the ratio 44:37, which, as he shows, approximates to $2^{1/4}$, and better approximations to π approach still nearer. I find it difficult to see how the demonstrations prove the proposition or to accept that the proposition justifies his using powers and roots of ratios. But let us overlook these difficulties and return to the "centres" of 1:2 and 2:3 which are to make up the difference between 3 and π.

These "centres" are not, as elsewhere in the treatise, approximations to ordinary square roots of a ratio, obtained by expanding the ratio into a discrete geometric proportion, the required root being the geometric mean, which lies between the harmonic and arithmetic means. Here Tartini takes his "centre" to lie between the arithmetic and contraharmonic means $\left(\frac{a+b}{2} \text{ and } a^2 + \frac{b^2}{a+b}\right)$; e.g. if 6, 8 (H.m.), 9 (A.m.), 10 (C.m.), 12, this kind of centre lies between 9 and 10, and not, as an ordinary geometric mean ($\sqrt{72}$), between 8 and 9. His justification for doing this is that [87]

> the quantity of the circumference is transcendental. Therefore its equivalent must be of the nature of a contraharmonic quantity, which corresponds to the negative quantity in Algebra, and by its own intrinsic nature transcends arithmetic unity.

I cannot make any guess at what this may mean.

This procedure is applied to the ratio 2:3, giving the ratio 25:26 (20, 24 (H.m.), 25 (A.m.), 26 (C.m.), 30), and to the square root of 1:2, i.e. $1:\sqrt{2}$ (presumably because of the above proposition about the *potenza dupla*), giving a ratio 36:37 (from an ap-

[86] Ibid., pp. 34-5.
[87] Ibid., p. 41: "la quantità della circonferenza è trascendentale. Dunque il ragguaglio dev'esser di natura di quantità contrarmonica, che corrisponde alla quantità negativa dell'Algebra, e per propria intrinseca natura tràscende la unità aritmetica".

proximation to $1:\sqrt{2}$, 5:7 expanded to 30, 35, 36, 37, 42). Tartini then, with no attempt at justification, takes the fourth root of this last ratio, 36:37, giving the approximation 289:291. The sum of the two ratios, 25:26 and 289:291 (i.e. these multiplied together), is then shown to approximate closely to π-3, and, as better approximations to π than $\frac{22}{7}$ are used, it approximates more closely [88].

There is a simpler, but no more convincing, demonstration of the intrinsic connexion between the ratio 1:3 and π, which Tartini repeats at the end of the book where he is trying to elucidate his circle-system [89]. He expands the ratio 1:3 in his usual way to 2, 3, 4, 6. He then argues that the diameter, which the circle has in common with the exscribed square, is arithmetic in nature, and so are the middle terms of the above series, being 3:4, the ratio of a fourth, which divides the octave arithmetically. The "centre", therefore, belongs to the diameter and is the arithmetic mean between 3 and 4, that is, $3\frac{1}{2}$. The circumference, being harmonic, can "only be the sum of the terms of the harmonic triple proportion", 2, 3, 6, that is, 11. Thus, doubling the terms, we have the ratio of diameter, 7, to circumference, 22. The most arbitrary step here seems to me to be the adding together of the terms of the harmonic proportion 2, 3, 6, an operation which Tartini describes in a defensively emphatic manner [90]:

> ciò, che appartiene alla circonferenza, come armonica, altro non può, nè dev'essere a tutto rigor matematico se non la somma della tripla armonica.

iv. The Mathematical Basis

II. The Circle and the Residue Series

An underlying assumption in both Tartini's treatises, necessary for the understanding of this section, is that the three kinds of proportion, Harmonic, arithmetic, geometric, are a descending

[88] Ibid., pp. 40-8.
[89] Ibid., pp. 43, 171-2.
[90] Ibid., p. 43.

hierarchy in respect of musical perfection and naturalness. We have already had examples of harmonic and arithmetic proportions hierarchized in this way; we must now add the geometric. Expressed schematically this hierarchy is [91]:

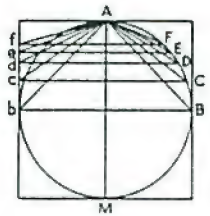
Harmonic proportion, rooted in the circle, produces the major triad, and the perfect cadence.
Arithmetic proportion, rooted in exscribed square, produces the minor triad, and the plagal cadence.
Geometric proportion, rooted in sines of the circle, produces the dissonances.

Tartini also quite often uses contraharmonic proportion, which, apart from the mysterious affirmation of its transcendental nature quoted above, seems to rank below harmonic, but has no direct musical results. The derivation of dissonance from geometric proportion is based on the fact that any two consonant intervals of the same kind added together produce a dissonant interval [92], e.g. two major thirds produce an augmented fifth, the three notes (C-e-g♯) forming the geometric series 1, $\frac{4}{5}$, $(\frac{4}{5})^2$. There is an unfortunate exception to this statement: the octave; this obliges Tartini to forbid in practice the use of a double octave (1, $\frac{1}{2}$, $\frac{1}{4}$), although he cannot of course claim that it is in fact dissonant.

In spite of his interest in the natural seventh, Tartini was, as I have said, determined to preserve Zarlino's *senario*, that is, the claim that all the consonances actually used in musical practice are contained in the harmonic series 1, $\frac{1}{2}$, $\frac{1}{3}$ . . . $\frac{1}{6}$. The problem is to give an adequate mathematical reason for stopping short of $\frac{1}{7}$ and not continuing indefinitely, as does the physical series of overtones. Zarlino, says Tartini, "has said many and fine things about the number six, but nothing conclusive" [93]; and Kepler, unknown to Tartini, had solved the problem simply and neatly by the circle

[91] Ibid., pp. 89-90.
[92] Ibid., pp. 73-4. This theory of dissonance is in Zarlino (see Shirlaw, op. cit., pp. 33-4). Vallotti (*Della Sc. Teor.*, pp. 92 seq.) also derives dissonances from geometric proportion.
[93] *Trattato*, p. 53: "sopra il numero senario ha detto cose belle, e molte, ma nulla concludenti".

and the heptagon. After many years of searching, Tartini has at last found the solution, also in the circle. His demonstration, very briefly summarized, is as follows [94]:



In the above figure the diameter, AM, is divided harmonically into $\frac{1}{2}$, $\frac{1}{3}$ . . . . $\frac{1}{6}$. By several, I think, arbitrary operations, Tartini derives from the chords AB, AC, etc. and the hypotenuses Ab, Ac, etc. a set of ratios: 1, 5:6, 3:4, 7:10, 2:3 (algebraically expressed, this series is derived from $\frac{1}{2}$, $\frac{1}{3}$ . . . . . $\frac{1}{n}$, by multiplying each term by $\frac{n+2}{2n}$). These, he claims are all consonant; but if we add the ratio deriving from the division of the diameter by $\frac{1}{7}$, it will be 9:14. This ratio is very near that of two major thirds added together, 16:25 (which exceeds 9:14 by only 224:225), and 16:25 is the third term in the geometric series, 1, 4:5, $(4:5)^2$. Geometric proportion is, as we have seen, the principle of dissonance. Therefore the addition of the term $\frac{1}{7}$ to the harmonic series would make it dissonant in nature; therefore it must be excluded, Q.E.D.

Apart from the arbitrary nature of the operations, this demonstration is self-contradictory. For in the series resulting from 1, $\frac{1}{2}$ . . . $\frac{1}{6}$ is the ratio 7:10, which can only be regarded as consonant if the natural seventh, introduced by $\frac{1}{7}$ in the harmonic series, is considered consonant; but the object of the demonstration is to show that the series must stop short of $\frac{1}{7}$. If 7:10 is not regarded as consonant, then Tartini's operations show that the series 1, $\frac{1}{2}$ . . . $\frac{1}{6}$, produces dissonances. Tartini skates quickly over this dilemma, but chooses the former horn, saying that by the ratio

[94] Ibid., pp. 56-9.

7:10 "the octave is determined into a consonant geometric system", i.e. 7:10 is to be considered consonant; the system is geometric because 7:10 is one of Tartini's approximations to $\sqrt{\frac{1}{2}}$, and would form with the diameter and radius the geometric series 1, $\sqrt{\frac{1}{2}}$, $\frac{1}{2}$.

The principle of confining consonant ratios to the series 1, $\frac{1}{2}$ . . . . $\frac{1}{6}$, has another weak point: it excludes the minor sixth, $\frac{5}{8}$. Tartini spends a long time dealing with this difficulty in the *Diss.*; and at the end one is left not knowing whether the minor sixth is to be considered a consonance or not [95]. But as regards the ratio 1:7, in this work Tartini, very wisely, merely affirms that $\frac{1}{7}$ is to be excluded from the series 1, $\frac{1}{2}$ . . . $\frac{1}{6}$, without giving any reasons [96]. But he gives some interesting hints of other reasons that he had up his sleeve [97]. Were it not that in this treatise, the *Diss.*, he had resolved to confine himself entirely to matters of fact, he would have revealed a hitherto unknown foundation of the six-system, namely, that 6 is a perfect number, being the sum of its factors, 1, 2, 3. The numbers making the basic harmonic ratios, octave 1:2 and fifth 2:3, also add up to 6, whereas the numbers of the basic geometric series, 1, 2, 4, add up to 7. He would have shown how "the nature of consonance cannot be separated from this basic principle of number". But since "this occult science that number contains in itself" is not now understood, he will not venture outside the field of generally accepted matters of fact. One can see here both the very strong attraction that Tartini felt towards numerology, as opposed to mathematics, and his sad realization that it would not be accepted by the enlightened century he was living in.

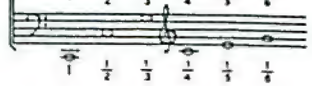
In the *Trattato* Tartini uses the circle also to provide a mathematical basis for the minor mode [98]. But, since he does not do so in the *Diss.*, and since the series of ratios he requires is so easily and directly derived from the harmonic series, I shall not (the

[95] *Diss.*, pp. 38-47.
[96] Ibid., pp. 12, 38.
[97] Ibid., pp. 47-9.
[98] *Trattato*, pp. 66-8.

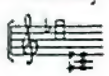
reader will be relieved to learn) expound his derivation of it from the circle. The series in question is formed by the residues of a string divided harmonically into $\frac{1}{2}, \frac{1}{3} \ldots \frac{1}{6}$, that is: $\frac{1}{2}, \frac{2}{3} \ldots\ldots \frac{5}{6}$,

or in musical notes [99]:

as compared with the harmonic series:

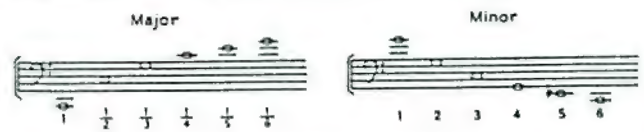


The harmonic series belongs to simultaneous harmony, all its intervals having the same *terzo suono*, the low C. The residue-series belongs to successive harmony, its intervals giving various *terzi suoni*: C (for C-g, C-e♮), F (for C-f), A♭ (for C-e♭). The former series is the basis of the major mode, which is more natural and perfect than the minor, based solely on the latter series. But the residue series contains both the major and minor triads, C-e♮-g and C-e♭-g, and is also essential to the major mode, as we shall see, since it contains the subdominant, F, not present in the harmonic series [100].

As we have already seen, Tartini justifies his use of the residue series on metaphysical grounds, and, at this point, he does so also on physical ones, apparently accepting Rameau's misleading experiment, which he later retracted. This experiment seemed to show that a string would set into sympathetic vibration strings a fifth and a major tenth below it,  the same notes as the *terzi suoni* produced by Tartini's residue-series [101]. But later in the *Diss.* he denies that the minor mode has any physical basis, as is shown by the harshly dissonant *terzo suono* A♭ produced by the minor triad  ; and this is another indication of the

[99] *Diss.*, pp. 60-4; cf. ibid., pp. 19-20, 25, 70-1, 108-9.
[100] *Trattato*, pp. 66-8.
[101] *Diss.*, p. 64. On Rameau's experiment, see Shirlaw, op. cit., pp. 202, 219-20; it was misleading because, although the strings F and A♭ are set in vibration by sounding the C above, they vibrate, not as wholes, but only in those parts which produce a C.

inferior, derivative nature of the minor mode as compared with the major. It is also a proof of the priority of mathematical or demonstrative principle over physical, since the residue-series is an essential basis also for the major mode, which, in the *terzo suono*, *has* a physical basis. "Therefore in reality from a demonstrative principle a law is given to a physical principle" [102].

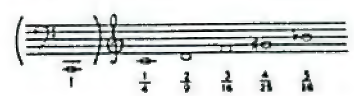
A simpler, more elegant derivation of the minor mode from the harmonic series, which Tartini gives in both treatises, is also evidently purely mathematical and without physical basis. It consists of making a mirror-image of the harmonic series by turning it into an arithmetic series (as always, in terms of frequencies the operation would be the other way round), thus:



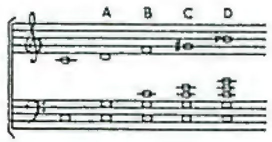
But Tartini prefers his residue-series, both because he can derive it from the circle, and because he needs it for his theory of cadences, the generation of the diatonic scale, and his fundamental bass.

In addition to the derivation of dissonances from geometric proportion, Tartini puts forward two other theories, one, appearing only in the *Trattato*, which he there accepts, and one, appearing only in the *Diss.*, which he rejects [102a].

The former theory is based on a series of notes which he derives from the sines of his circle [103]. Again, as Tartini himself remarks, this is so easily derived from the residue-series, namely, by squaring the denominators, that I will not bother the reader with the circle. It is (1), $\frac{1}{4}, \frac{2}{9} \ldots\ldots \frac{5}{36}$



[102] *Diss.*, pp. 108-9: "Adunque realmente da un principio dimostrativo vien data legge ad un fisico principio".
[102a] *Trattato*, p. 66; *Diss.*, pp. 19-20. This is also in Zarlino (see Shirlaw, op. cit., p. 35).
[103] *Trattato*, pp. 50, 60, 73-4.

He then puts these notes over chords built from the harmonic series on the same fundamental:

thus obtaining dissonant chords, all containing two consonant intervals of the same kind: two fifths at A, two fourths at B, two major thirds at C, two minor thirds at D. This both explains and proves the principle that dissonance is founded on geometric proportion, since, e.g. the two fifths form the proportion 1, $\frac{3}{2}$, $(\frac{3}{2})^2$.

The very interesting and attractive theory in the *Diss.*[104] arises from a discussion of the minor seventh G-f in C major, i.e. the dominant, seventh, which in Ptolemy's just scale has the ratio 9:16[105]. This seventh is[106]

> so homogeneous that it is used in practice with quite peculiar privileges, which, even if they do not make it a positive consonance, certainly single it out as being half-way between the consonances and dissonances.

This uneasy position of the dominant seventh has produced great confusion in the minds of musical theorists, and Tartini will now examine one of the erroneous theories which have arisen from it. I suspect that Tartini made up this theory himself and that, although he rightly rejected it as having at least one irremediable defect, he was quite strongly attracted by it.

The theory rests mathematically on a harmonic series having only odd denominators: 1, $\frac{1}{3}$, $\frac{1}{5}$ . . . . . $\frac{1}{15}$. Just before his refutation of it, Tartini had claimed that the only true diatonic dissonances (i.e. not involving accidentals) are the following black notes:

(the very odd inclusion of C-a as a dissonance is not explained).

[104] *Diss.*, p. 89.
[105] Cf. supra p. 139.
[106] Diss., pp. 88-9: "é dissonanza talmente omogenea, che si usa in pratica con privilegj affatto particolari, i quali se non la determinano consonanza positiva, certamente la singolarizzano mezzana tra le consonanze, e le dissonanze."

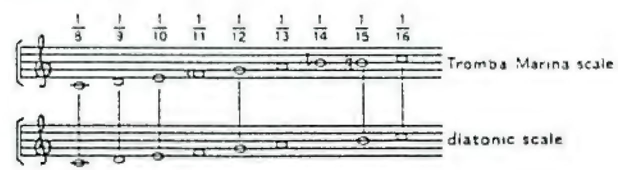
Now we can say that

the dominant 7th C-b♭, 9:16,	is near to 4:7	(difference: 63:64)
the ninth, C-d,	is exactly 4:9	
the eleventh, C-f, 3:8,	is near to 4:11	(difference: 32:33)
the thirteenth, C-a, 3:10,	is near to 4:13	(difference: 39:40)
the fourteenth, C-b♮,	is exactly 4:15	

Taking the whole series 1, $\frac{1}{3}$, $\frac{1}{5}$ . . . . $\frac{1}{15}$, and ignoring the interpolated octaves, we can therefore say that the first three terms give the exact ratios for the basic consonances (C, G, E), the fourth term is the half-way minor seventh (either approximately the dominant seventh, or exactly the natural seventh), and the last four terms give Tartini's four diatonic dissonances. This makes a very tidy scheme:

$$\underbrace{1 \quad \tfrac{1}{3} \quad \tfrac{1}{5}}_{\text{basic consonances}} \qquad \underset{\text{half-way minor 7th}}{\tfrac{1}{7}} \qquad \underbrace{\tfrac{1}{9} \quad \tfrac{1}{11} \quad \tfrac{1}{13} \quad \tfrac{1}{15}}_{\text{diatonic dissonances}}$$

Tartini then examines the physical basis of this theory as exemplified in practice on the tromba marina, "vero fisico monocordo di natura"[107], by comparing the scale played on this instrument with the ordinary diatonic scale:



Only the notes joined by a line are of exactly the same pitch. The tromba marina scale contains all the above dissonances, since $\frac{1}{14}$ is merely $\frac{1}{7}$ plus an octave, and with this natural seventh it introduces an extra note into the diatonic scale. This interval, 1:7, really does occur naturally on this instrument and is an addition to the "successive consonant harmony, which not only brings

[107] Ibid., pp. 90-1; cf. ibid., pp. 105-7.

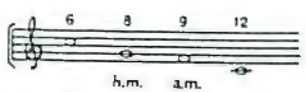
with it an appearance of truth, but is a positive truth" [108]. It is in this context that Tartini suggests adding the natural seventh to his fundamental bass for the diatonic scale [109]. But, in spite of this favourable preamble, he rejects both the use of the natural seventh and the whole 1, $\frac{1}{3}$ . . . $\frac{1}{15}$ theory for two reasons, of which the second is certainly valid.

First [110], if we accept B♭ into the diatonic scale as making a consonant chord C-e-g-b♭, we must also accept analogous chords on the dominant and subdominant, G-b♮-d-f♭ and F-a-c-e♭. This would add two more notes to the scale, F♭ and E♭, neither of which occurs in the tromba marina scale and which therefore have no physical basis. Second [111], the series 1, $\frac{1}{3}$ . . . $\frac{1}{15}$ does not in fact give the essential notes F and A of the diatonic scale, as one can see from the tromba marina scale, in which F is too sharp by 32:33 and A too flat by 39:40, both very considerable differences.

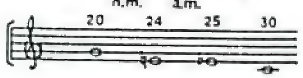
v. The Musical Basis

We must now return to Tartini's residue-series, from which he derives not only the major and minor modes, as we have seen, but also the diatonic scale and his three kinds of cadence [112]. This series gives us two discrete geometric proportions:

(i) the octave divided harmonically and arithmetically



(ii) the fifth divided harmonically and arithmetically



(ii), as we have seen, contains the major and minor triads. (i) contains the tonic, dominant and subdominant, the bases of Tartini's cadences, and from major triads on these notes Tartini derives the major diatonic scale. This derivation of the scale does in fact give the same succession of major and minor tones and of semitones as Ptolemy's syntonon, the just scale advocated by the great majority of theorists from the sixteenth century onwards. And Tartini is surely correct in his assertion that scales derive from harmony or the basic consonances, octave, fifth and fourth, and not *vice versa*, even the scales of the ancient Greeks, which are different ways of filling up the primary datum, the tetrachord. But it is strange that, in the *Diss.*, after he had read at least some of Rameau's works, he should express astonishment and bitter disappointment that his derivation of the scale, "una verità si luminosa, e importante", which he had already expounded in the *Trattato*, published many years before, should have produced no response whatever "in un secolo sì illuminato, qual si chiama il presente" [113]. It is strange because one of Rameau's basic principles, in all his works, is that melody derives from harmony and not the other way round, and, in his *Génération Harmonique*, published in 1737, seventeen years before the *Trattato*, he had derived the just diatonic scale from the triads on the tonic, dominant and subdominant [114].

Tartini also claims that this theory, based on the harmonic and arithmetic divisions of the octave and fifth, has remained unknown since the time of Plato and Pythagoras. Here he must mean his derivation of these divisions from the residue-series and not the divisions themselves, which already play an important part in the harmonic doctrine of Zarlino, whom Tartini certainly had read [115]. But it is, I think, true that Tartini is original in using the twofold division of the octave as the basis of his system of cadences, and that he is correct in denying that the subdominant can ever be derived from the harmonic series—Rameau was forced to extraordinary lengths of mistaken ingenuity in trying to do so [116].

Tartini's system has three kinds of cadence, in a descending order of perfection and finality [117]: perfect (*cadenza armonica*),

[108] Ibid., p. 91: "successiva consonante armonia, che non solamente porta seco apparenza di verità, ma è una verità positiva".
[109] V. supra p. 142.
[110] *Diss.*, pp. 92-3.
[111] Ibid., pp. 95-6.
[112] *Diss.*, pp. 66-7, 71-2, 75; *Trattato*, p. 98.

[113] *Diss.*, pp. 71-2.
[114] See Shirlaw, op. cit., p. 187.
[115] *Diss.*, p. 67; on Zarlino, see Shirlaw, op. cit., pp. 47-50.
[116] See Shirlaw, op. cit., pp. 265-6, 270-1.
[117] Tartini, *Trattato*, pp. 102-3.

dominant to tonic; plagal (*cadenza aritmetica*), subdominant to tonic; and "mixed" (*cadenza mista*), subdominant to dominant. A cadence, he says, is analogous to a period in speech, and he describes the effect of the three kinds as follows. The perfect cadence is "of a strong, majestic and lively harmony"; the plagal is "of a languid and sweet harmony"; the mixed is "of a sustained and not entirely determined harmony; like an exclamation mark in speech". The first two kinds of cadence present us with no problem; but the third, the *cadenza mista*, is one of the most baffling of all the many puzzles in Tartini's musical theory. There never has been any such cadence; and, as Tartini himself points out when discussing his fundamental bass to the diatonic scale, the progression subdominant to dominant is an awkward one, in which it is difficult to avoid consecutive fifths and octaves. Moreover, when the progression does occur in ordinary classical harmony, it is plainly not a cadence itself but part of the perfect cadence subdominant-dominant-tonic, as in Tartini's harmonization of the scale. But on this extremely frequent cadential bass, F-G-C in C major, it is much more usual to find on the F not the plain subdominant but the chord of the added sixth (F-A-C-D) or $\frac{6}{5}$ of supertonic, and of course on the G not the plain dominant but either the dominant seventh or a $\frac{6}{4}$ chord going to a $\frac{5}{3}$. The only, and not very satisfactory explanation I can suggest is that the *cadenza mista* is a distorted version of Rameau's "irregular" cadence [118], that is, either tonic to dominant (imperfect cadence or half-close), which is extremely common in eighteenth-century music, or subdominant to tonic, in the latter case the penultimate chord usually having an added sixth. But it remains quite extraordinary that in both his treatises Tartini should introduce, without a word of explanation, a non-existent and impossible cadence.

Tartini's system of cadences points to a basic defect in his general treatment of practical harmony, namely, his extremely archaïc conception of the use of dissonances, especially if one compares it with Rameau's remarkable insights into the function

[118] See Shirlaw, op. cit., pp. 111-2.

of dissonance in determining progressions and cadences and in defining tonality. The rules Tartini gives, in both treatises [119], for the use of dissonances might come straight out of Zarlino, and probably did; they codify the practice of a very conservative composer of the mid-sixteenth century. All dissonances must be prepared and resolve downwards by step, that is, must be suspensions. There is no mention of upward resolutions, very common by the middle of the eighteenth century and obligatory for the chord of the added sixth, nor of *appoggiature* or accented passing-notes. The only mention of a dissonant note resolving on to a new bass is the casual remark that the seventh sometimes does this, "of which at present there is no point in assigning a cause" [120]; this is most disappointing in view of his interesting remarks on the dominant seventh in connexion with the natural seventh. All he has to say about the $\frac{6}{4}$ chord, the focal point of later eighteenth-century harmony, is that "there are not many cases in which it can be used with good effect" [121]. All this is most odd and unexpected in a theorist who was also an experienced performer and composer. Again, I can suggest only a partial explanation, namely, that in Tartini's threefold hierarchy of *scienze* the third, concerned with musical practice, occupied in his mind such a low position with respect to the mathematical and physical kinds of knowledge that his only interest in it was to make it conform with, and confirm the two higher kinds. But this in itself is of course strange for a man who has devoted a long life to the practice of music.

The same overriding interest in the higher categories of knowledge, and also the ultimate priority of mathematical truths, is evident in Tartini's discussions of temperament. In the *Diss.*[122] these occur in the context of his refutation of the derivation of diatonic dissonances from the series 1, $\frac{1}{3}$ . . . . $\frac{1}{15}$, which he had

[119] *Trattato*, pp. 78-82; *Diss.*, p. 104.
[120] *Trattato*, p. 78: "di che nel caso presente nulla importa l'assegnare la cagione".
[121] *Trattato*, p. 105: "non molti sono que' casi, ne' quali si possa usare con buon' effetto".
[122] *Diss.*, pp. 95-6.

rejected because F and A, $\frac{1}{11}$ and $\frac{1}{13}$, are badly out of tune, the former being too sharp and the latter too flat. It might be argued, however, that good players of the tromba marina, or horn or any other instrument producing the harmonic series, adjust these notes so that they are very near to, or the same as a tempered fourth C-F and a tempered sixth C-A (the former being a little wider than a just fourth and the latter a little wider than a just sixth). This argument, says Tartini, is invalid because temperament is posterior to the physico-mathematical principles by which it can be demonstrated that no harmonic series based on C can ever produce a true F or A. The argument certainly is faulty because it is self-contradictory to try to explain actual harmonic practice by a system of ratios physically present in some instruments, and then drastically alter those ratios.

But one might use an argument from temperament to undermine the whole of Tartini's system (and Rameau's). Since all the intervals of the diatonic scale, except the octave, are in practice altered by temperament,

> In this universal invasion of temperament are the three cadences, as cadences, included? Their order with respect to their greater or less perfection? The same number of eight musical notes arranged in the cadences as in the diatonic scale? the fundamental formula of the major and minor modes? No; they are not, because temperament has nothing to do with their intrinsic nature.

This intrinsic nature is based on mathematical (*dimostrativo*) principle, which is

> positive and real and certainly prior by nature to the physical principle . . . Temperament alters physically all the actual ratios of the diatonic intervals except the octave. It does not alter in any way their original denomination, the altered fifth remains 2:3, the altered fourth remains 3:4, as much as the unaltered octave remains 1:2.

And so the whole system of cadences, modes, scale etc. still stands [123]. Tartini's reasoning here is nearer to a declaration of

[123] Ibid., pp. 109-110: "In questa universal invasione di temperamento sono poi incluse le tre cadenze come cadenze? L'ordine delle medesime a ragguaglio della

faith than to logical argument, but, I think, necessarily so. He is discussing the basic presuppositions of his system, its metaphysical foundation, and metaphysical statements cannot be proved or disproved; they can only be accepted or rejected. But it is possible, as with the existence of God, to back up one's confession of faith by an appeal to the *consensus gentium*, and this Tartini does. The diatonic genus rests on the mathematical system of untempered intervals, and this genus "has been impressed by nature on the universal feeling of the human race [124].

In the *Trattato* [125], Tartini begins by admitting the necessity of temperament and its universal use, since singers and instrumentalists have to model their intonation on that of the *basso continuo*, the organ or harpsichord, which, unless the number of keys is unduly multiplied, can play only the octave in tune. This is bad enough, but what is worse is that there is no standard system of temperament, but many different competing ones. There is no hope of imposing any one system as more reasonable than the others, because "il pretendere di diformar le ragioni con ragione è contraddizione manifesta". This is a case for prudence rather than theoretical reasoning. Tartini then accepts wholeheartedly the extremely interesting solution to the problem proposed and put into practice by his friend and colleague Padre Vallotti, a distinguished organist and composer as well as theoretician. Vallotti tuned the white notes of his organ in just intonation and let all the errors accumulate in those black notes that are required only in more remote keys, which are in practice very little used. What is meant, I think, is that, for example, F♯ and B♭ were tuned

loro maggior, o minor perfezione? Il numero identifico di otto note musicali si nelle cadenze ordinate, che nella scala diatonica? La formola fondamentale dei due modi maggior, e minore? Nò: non vi sono, perchè alla loro intrinseca natura nulla appartiene il temperamento . . . . un principio positivo, e reale, e certamente prior di natura del fisico principio . . . Il temperamento altera fisicamente tutte le attuali ragioni degl'intervalli diatonici fuorchè l'ottava. Non altera in modo veruno nè loro original denominazione (la quinta alterata resta sesquialtera: la quarta alterata resta sesquiterza ec., quanto la ottava non alterata resta dupla,) nè le cadenze . . . . ."

[124] Ibid., p. 110: "che dalla natura si trova impresso nel sentimento universale della umana specie".

[125] *Trattato*, pp. 99-100.

as just major thirds to D, and G♭ and A♯ were therefore considerably out of tune. This system had the advantage of producing a real effect of key-colour. Vallotti pointed out the pleasure which results

> from the contrast of the greater and less perfection of the chords in the various modulations occurring. If the temperament were equal, or more or less so, there would certainly not be this *chiaro oscuro* which in practice produces an excellent effect [126].

Tartini then goes on to say that on the violin he is able to play in just intonation, with the help of the *terzo suono*, and that he teaches his students to do so—an important statement, since generations of professional string-players must have passed through his hands.

vi. Conclusion

I have tried in this chapter to do two things: first, to show, as others have done in different ways, the importance and originality of Tartini's musical theory, and, second, to point out, as no one has yet done, some examples of just where and how his system becomes senseless and arbitrary. In attempting this latter, critical aim I have necessarily had to mention his archaisms and his errors about physical facts; but these are not my main concern, but rather to pick out the weak points in the structure of his whole system. By "weak points" I mean not only what is illogical or erroneous, but also what is untidy, botched, aesthetically unsatisfactory. I can perhaps make this aim clearer by briefly comparing Tartini's system with Kepler's [127], which, although also not without its weak spots, is on the whole much stronger and more convincing.

[126] Ibid., p. 100: "dal confronto della perfezione maggiore, e minore degli accordi rispetto alle varie modulazioni, che occorrono. Se il temperamento fosse eguale, o poco più poco meno, non vi sarebbe certamente questo chiaro oscuro, quale in pratica produce ottimo effetto."

[127] Benjamin Stillingfleet, in his *Principles and Power of Harmony* (London, 1771, pp. iv-v, 17), notes the likenesses between Kepler and Tartini, both of them deriving from the same Pythagorean-Platonic tradition. Vallotti (*Della Sc. Teor.*, pp. 59, 60-1) dismisses both Kepler's theory of consonance and Tartini's circle-theories in the same words: "una pura e mera analogia".

The main strength of Kepler's musical theory [128] is that it forms part of an all-embracing metaphysical, religious and cosmological system, centred on his God ἀει γεωμετριζων, who has created the universe on geometric archetypes and has implanted these in man's soul. If, for the purpose of understanding Kepler, we accept this God, then the order and coherence of the whole system, which describes His creation and image, upholds any particular part of it, in this case, Kepler's derivation of musical consonances from the regular polygons inscribed in a circle and the other speculations based on this derivation. Now, in Tartini's published works, this metaphysical-religious framework is entirely lacking; one can only infer, as I have tried to show, the priority of mathematical knowledge over other kinds, and guess at some Platonic God to account for this priority. But from his letters to Martini, and from the extracts from his *Scienza Platonica* given by Capri, it is evident that Tartini considered his musical and mathematical system as a divine revelation, which was destined, by demonstrating the purely mathematical structure and causes of the universe, to defeat the mechanistic and atheistic science of the *philosophes*, *i Dotti* [129]. But even if Tartini had been able to publish the religious background to his system (which, in eighteenth-century Padua, is most improbable), this would not have given it the strength and beauty of Kepler's, for the following reasons.

Though both systems derive ultimately from the *Timaeus* and both are founded on the circle and its properties, they differ fundamentally in that Tartini's is arithmetical, expressed entirely in numbers and explicitly taking number as the highest mathematical reality [130], whereas Kepler's is geometric and numbers are given a low metaphysical status. Both thinkers refuse to use algebra. For Kepler, since his main demonstrations and analogies are geometric, this matters much less than for Tartini, all of whose

[128] V. supra Ch. IV.

[129] See Capri, op. cit., p. 498.

[130] Tartini, *Diss.*, pp. 29-31; cf. Capri, op. cit., pp. 497, 501-2.

arguments, in so far as they are valid universal propositions [131], would be enormously clearer if expressed algebraically. The dense obscurity resulting from Tartini's numerical presentation of his system is important, not only because it prevented his contemporaries from even trying to understand him, but also because it produced confusion and delusion in his own thinking. At the core of his demonstration that the circle is intrinsically harmonic is the identity [132]: harmonic mean times arithmetic mean equals geometric mean squared ($H \times A = G^2$), which seems to him a discovery of staggering importance. Now, if one thinks of ratios solely in numbers, this identity would be by no means obvious nor easy to discover; it is only if one expresses it algebraically that it becomes self-evident and trivial:

$$\frac{2ab}{a+b} \times \frac{a+b}{2} = ab$$
$$(H \times A = G^2)$$

Kepler's God created the universe on the geometric model, coeternal with Him, of the regular plane and solid figures; Tartini's God created it in number, by combining the three proportions, harmonic, arithmetic, geometric. In consequence, Kepler's system, though extremely complicated in its details, is beautifully simple in its general plan: the plane figures give the ratios exhibited in polyphonic music and in the varying velocities of the planets, and the solid figures, which derive from the plane, give the distances between the planetary orbits. Tartini's system, clogged with arithmetical calculations, painfully produces, at best, true but trivial statements, such as the above identity, and, at worst, hopelessly arbitrary juggling with numbers, such as the attempts to show that 1, 2, 2π is some kind of harmonic triad. Where Tartini's system is tidy and logical, as in the derivation

[131] With one exception (*Trattato*, pp. 49-50), Tartini's propositions are, I think, genuinely universal; and Rousseau's criticism of him on this score, in the otherwise favourable account of his system (Rousseau, *Dictionnaire de Musique*, Amsterdam, 1768, art. Système), is unwarranted (cf. *Risposta di un Anonimo al celebre Sig. Rousseau* . . . , Venezia, 1769).

[132] V. supra p. 148.

from harmonic, arithmetic and geometric proportion of, respectively, major and minor consonances and dissonances, it is not original and, as concerns dissonances, is musically archaic and unenlightening. Where Kepler's system is illogical and fanciful, as in his sexual analogies, drawn from the Golden Section and the Fibonacci numbers and applied to thirds and sixths, it is musically illuminating and, in a poetic way, satisfying.

The obscurity of Tartini's writings is certainly not only due to his rejection of algebra. As I have already mentioned, he himself accounts for it in two different ways. First [133], in his published works, the obscurity is said to arise necessarily from the extreme novelty and inherent complexity of his system. But he also repeats this explanation in the *Scienza Platonica*, and moreover welcomes "the extreme difficulty of this science", which confines it to a few chosen intellects, thanking God for having thus prevented His precious gift from becoming too commonly known. Second [134], in his letters to Martini, in the *Scienza Platonica*, and at one place in the published *Risposta*, he admits to reinforcing this inevitably esoteric character of his new science by deliberate obscurity. He is surrounded by impious materialists who will stop at nothing to suppress, distort or ridicule his system, once they realize that it will utterly refute them. The message, therefore, must be disguised so as to reach only suitable ears. Moreover, some parts of it are so extraordinary that they must be conveyed only by personal interview and be confined to a very few close friends. After his death others will speak of matters which cannot now be made public, but which are "essentially dependent on this science".

Perhaps when all Tartini's unpublished works and letters have been studied we shall know what these extraordinary and important secrets were. At present, I think one can safely do no more than conjecture that they were of a religious nature. Tartini was deeply convinced of having been favoured with a divine revelation

[133] V. supra, p. 145-6.

[134] Capri, op. cit., pp. 419, 449, 496; *Risposta*, p. 7.

and acutely conscious of the apparent absurdity of an uneducated professional violinist claiming to have invented a new mathematical science—but God chooses as His instruments the humble and meek in order to confound the pride of the wise [135]:

> If God for His greater glory wishes to use the jawbone of an ass (which is what I am) in order to confound the pride of others, are we to suppose that the jawbone of an ass will not produce the effect intended by God?

Near the beginning of his *Risposta* Tartini again emphasizes the apparent absurdity of his claim, and concludes [136]:

> Either the Author must consider himself the most solemn fool on earth; or the learnèd mathematical world is bound to reflect with the greatest possible earnestness on this matter.

I certainly do not think that I have in this chapter demonstrated the first of these alternatives, and I have every hope that other scholars, more erudite and more competent in mathematics than I, will make better sense of Tartini's system than I have been able to do. But I strongly suspect that a residue of madness and nonsense will always remain.

[135] Martini, *Carteggio*, pp. 334-7: "Se Iddio per sua maggior gloria vuol adoprare una mascella d'Asino (e son io) per confonder la superbia altrui, teneremo forse, che la mascella d'Asino non faccia l'effetto propostosi da Dio?".

[136] *Risposta*, pp. 7-9: "O l'Autore deve tenersi per il pazzo più solenne della Terra; o il dotto matematico Mondo è costretto a rifletter con l'ultima serietà su'l caso presente".

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